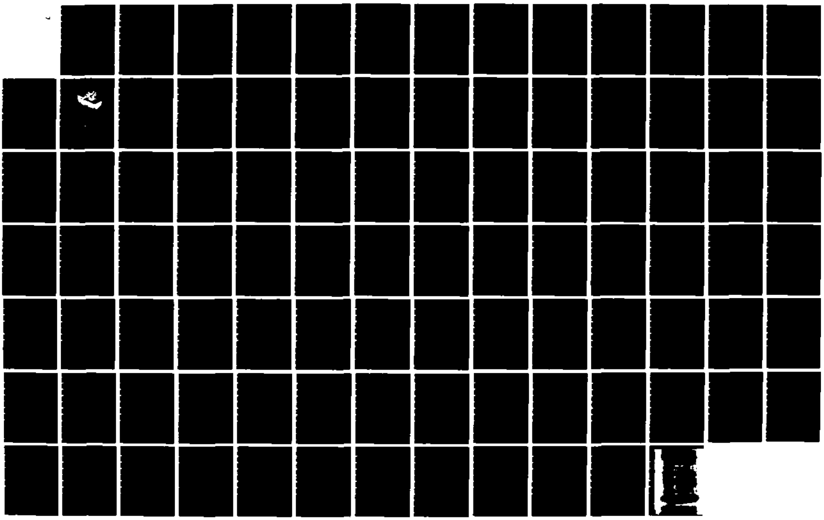
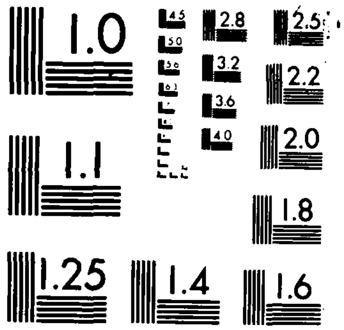
AD-A141 797 MATHEMATICAL PROGRAMMING FOR AIR DEFENSE COMMAND AND CONTROL(U) ARMY MILITARY PERSONNEL CENTER ALEXANDRIA VA B E SMITH 11 MAY 84 1/1

UNCLASSIFIED F/G 12/1 NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963 A

AD-A141 797

MATHEMATICAL PROGRAMMING FOR AIR DEFENSE COMMAND AND CONTROL

CPT Brian E. Smith
HQDA, MILPERCEN (DAPC-OPA-E)
200 Stovall Street
Alexandria, VA 22332

Final report 11May 1984

Approved for public release ; distribution unlimited

A thesis submitted to the Massachusetts Institute of Technology, Cambridge, Massachusetts in partial fulfillment of the requirements for the degree of Master of Science in Operations Research.

84 06 04 040

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) MATHEMATICAL PROGRAMMING FOR AIR DEFENSE COMMAND AND CONTROL		5. TYPE OF REPORT & PERIOD COVERED Final Report 11 May 84
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) CPT Brian E. Smith U.S. Army		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Student, HQDA, MILPERCEN (DAPC-OPA-E), 200 Stovall Street, Alexandria, Virginia 22332		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS HQDA, MILPERCEN, ATTN: DAPC-OPA-E, 200 Stovall Street, Alexandria, Virginia 22332		12. REPORT DATE 11 May 1984
		13. NUMBER OF PAGES 90
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release ; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

This thesis was submitted to the Department of Electrical Engineering and Computer Science in partial fulfillment of the degree of Master of Science in Operations Research at the Massachusetts Institute of Technology.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Mathematical Programming, Analytic Modeling, Linear Programming, Integer Programming, Network Display, Dynamic Programming

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report illustrates the advantages of using the techniques of mathematical programming during an Air Defense exercise. It is directed toward the optimal deployment of Air Defense resources. Three levels of command in the Army Air Defense structure are involved in this study: the Group, the Battalion, and the Battery. Analytic modeling, linear and integer programming, network theory, and dynamic programming are used to develop this

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Block 20.

approach for improved command and control at two of these levels. The immediate benefit of this approach is a reduction in personnel requirements in the command center, shorter response time, and optimal allocation of very scarce Air Defense resources. By applying very basic techniques of mathematical programming, the Army Air Defense field operations can be considerably improved.

MATHEMATICAL PROGRAMMING FOR

AIR DEFENSE COMMAND AND CONTROL

by

BRIAN E. SMITH

B.S., United States Military Academy (1977)

M.S., University of Southern California (1980)

SUBMITTED TO THE DEPARTMENT OF
ELECTRICAL ENGINEERING AND COMPUTER SCIENCE
IN PARTIAL FULFILLMENT OF THE
DEGREE OF

MASTER OF SCIENCE IN
OPERATIONS RESEARCH

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 1984

Signature of Author ....
Department of Electrical Engineering and Computer Science
11 May 1984

Certified by ....
Jeremy F. Shapiro
Thesis Supervisor

Accepted by ....
Richard C. Larson
Co-Director, Operations Research Center

MATHEMATICAL PROGRAMMING FOR AIR DEFENSE COMMAND AND CONTROL

by

BRIAN E. SMITH

Submitted to the Department of Electrical Engineering and Computer Science on 11 May 1984 in partial fulfillment of the requirements for the Degree of Master of Science in Operations Research

ABSTRACT

This report illustrates the advantages of using the techniques of mathematical programming during an Air Defense exercise. It is directed toward the optimal deployment of Air Defense resources rather than any new methods of weapon system operations. Three levels of command in the United States Army Air Defense structure are involved in this study: the Group, the Battalion, and the Battery. Analytic modeling, linear and integer programming, network theory, and dynamic programming are used to develop this new approach for improved command and control at two of these levels. The immediate benefit of this approach is a reduction in personnel requirements in the command center, shorter response time, and optimal allocation of very scarce Air Defense resources. By applying very basic techniques of mathematical programming, the Army Air Defense field operations can be considerably improved.

Thesis Supervisor: Dr. Jeremy F. Shapiro

Title: Professor of Operations Research and Management

ACKNOWLEDGEMENTS

I would like to thank my family for their continuous support and confidence, especially Patty, Patricia Anne, and Brian Andrew, who started and finished this program with me.

I would also like to thank my thesis advisor, Professor Shapiro, for his guidance, my course advisor, Professor Larson, for his patience, my writing critic, Ilse S. Levis, for her helpful comments, and my typist, Kathleen Sumera, for her typing expertise.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

CHAPTER 1

INTRODUCTION

Command and control procedures for Short Range Air Defense (SHORAD) weapon systems during a field exercise are inefficient in terms of time, personnel, and equipment, and ineffective in providing the commander with the most accurate information available for making optimal or near optimal decisions.

The objective of this paper is to illustrate how the techniques of mathematical programming, including analytic modeling, network theory, linear and integer programming, and dynamic programming, can provide the Air Defense commander with a valuable decision-making tool during an Air Defense exercise.

Mathematical programming techniques can insure the optimal allocation of limited Air Defense resources, optimal scheduling of weapon system's defense assignments, reduction of staff personnel in the command centers, and rapid redesign of defense coverage, based on the latest unit status, throughout the exercise.

The results of this study indicate that mathematical programming techniques can effectively be applied to military operations and should be used as a decision-making tool for command and control during a SHORAD field exercise. Similar approaches may also benefit other branches of the Army and departments of the Armed Forces involved with coordinating multiple levels of command, illustrating extensive details during an exercise, and effectively and efficiently allocating and scheduling military resources.

CHAPTER 2

CURRENT METHOD OF AIR DEFENSE OPERATIONS

The current method of Air Defense operations is discussed at three levels of command: the Group, the Battalion, and the Battery level. Each level has a different mixture of weapon systems to plan with and deploy, a unique command post structure in the field environment, and diverse methods of operating during an exercise.

2.1 Levels of Command

2.1.1 Battery Level:

The lowest command level is that of the Battery. It consists of approximately 150 soldiers in four weapon system Platoons and one headquarters Platoon. Each Platoon has four primary weapon systems, and two of the Platoons are usually assigned one or two Redeye teams. A typical composite Chaparral/Vulcan Battery would have a total of eight Chaparral weapon systems, eight Vulcan gun systems, two to four Redeye systems, and an assortment of support vehicles. A "pure" Chaparral or Vulcan Battery would have a total of 12 weapon systems, Chaparral or Vulcan, plus the assigned Redeye systems.

2.1.2 Battalion Level:

The Battalion command level is the next in line up the hierarchy in Air Defense. It consists of one headquarters Battery, three composite Batteries or four pure Batteries, and a Signal Platoon for communications support. An important element of command and control, the staff, is included in the headquarters Battery, but typically identifies with individual staff sections such as: administration, intelligence, plans and operations, logistics, maintenance, and communications. A typical Battalion consists of 24 Chaparrals, 24 Vulcans, 6 Redeye teams, and numerous support vehicles.

2.1.3 Group Level:

The Air Defense Group consists of an officer and senior non-commissioned officer-heavy headquarters Battery, three Chaparral/Vulcan Battalions, a Hawk Battalion, and a Signal Company (equal in size to a battery). It usually covers a very large area of defense, including several airfields, storage sites, and supply depots. The command and control elements at this level are divided into large staff sections with a variety of responsibilities, including administration and personnel, military intelligence and security, communications, plans and operations, logistics, finance, and maintenance. It becomes apparent that the higher up the chain-of-command levels, the more difficult it becomes to organize and conduct a coordinated field exercise.

2.2 Weapon Systems

The Air Defense weapon systems considered in this study are the Chaparral guided missile system (Figure 2.1), the Vulcan gun system (Figure 2.3), and the Redeye guided missile system (Figure 2.2). A description of each weapon system follows, in conjunction with current unclassified Army Field Manuals.

2.2.1 Chaparral:

The Chaparral weapon system is a highly mobile surface-to-air missile system designed to counter the high-speed, low-altitude air threat to critical assets in the forward areas of the battle-field. Chaparral is

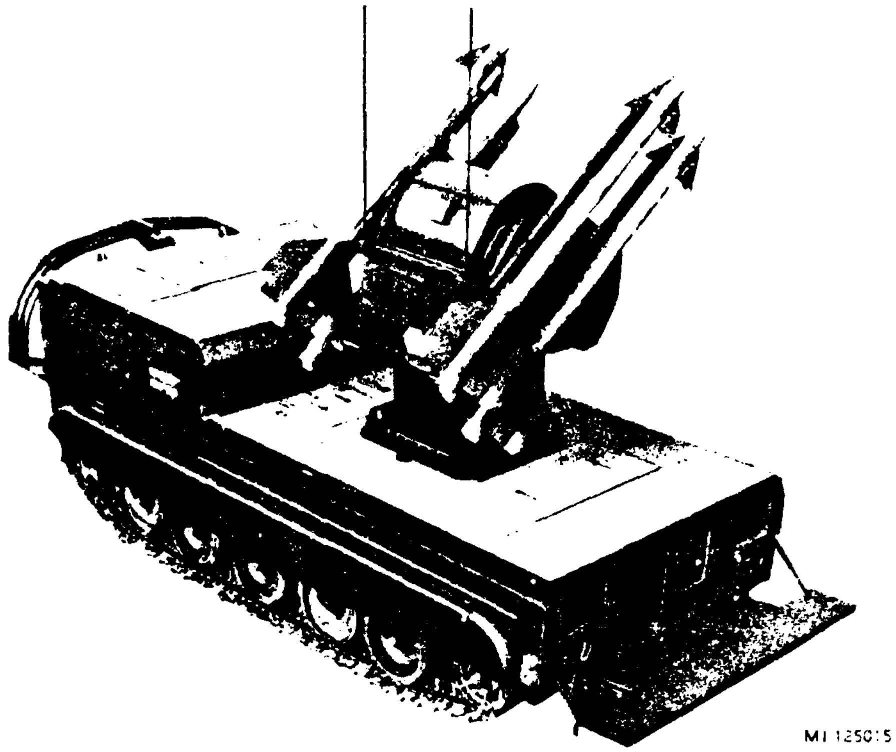


Figure 2.1
Chaparral Guided Missile System

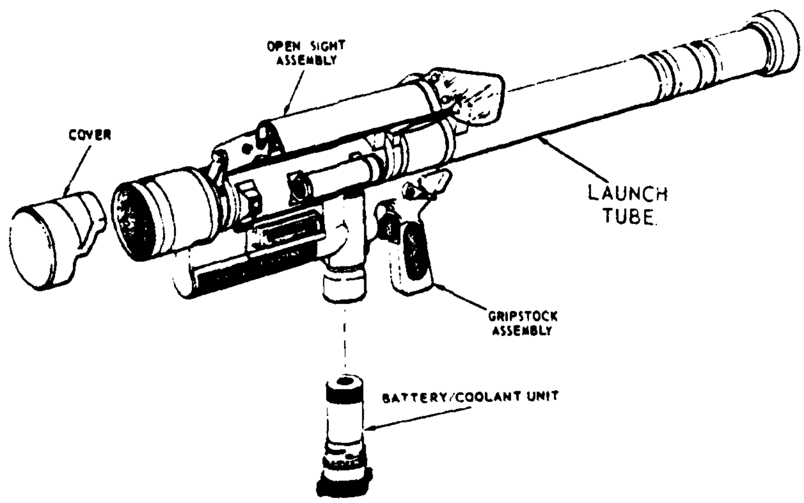


Figure 2.2
Redeye Guided Missile System

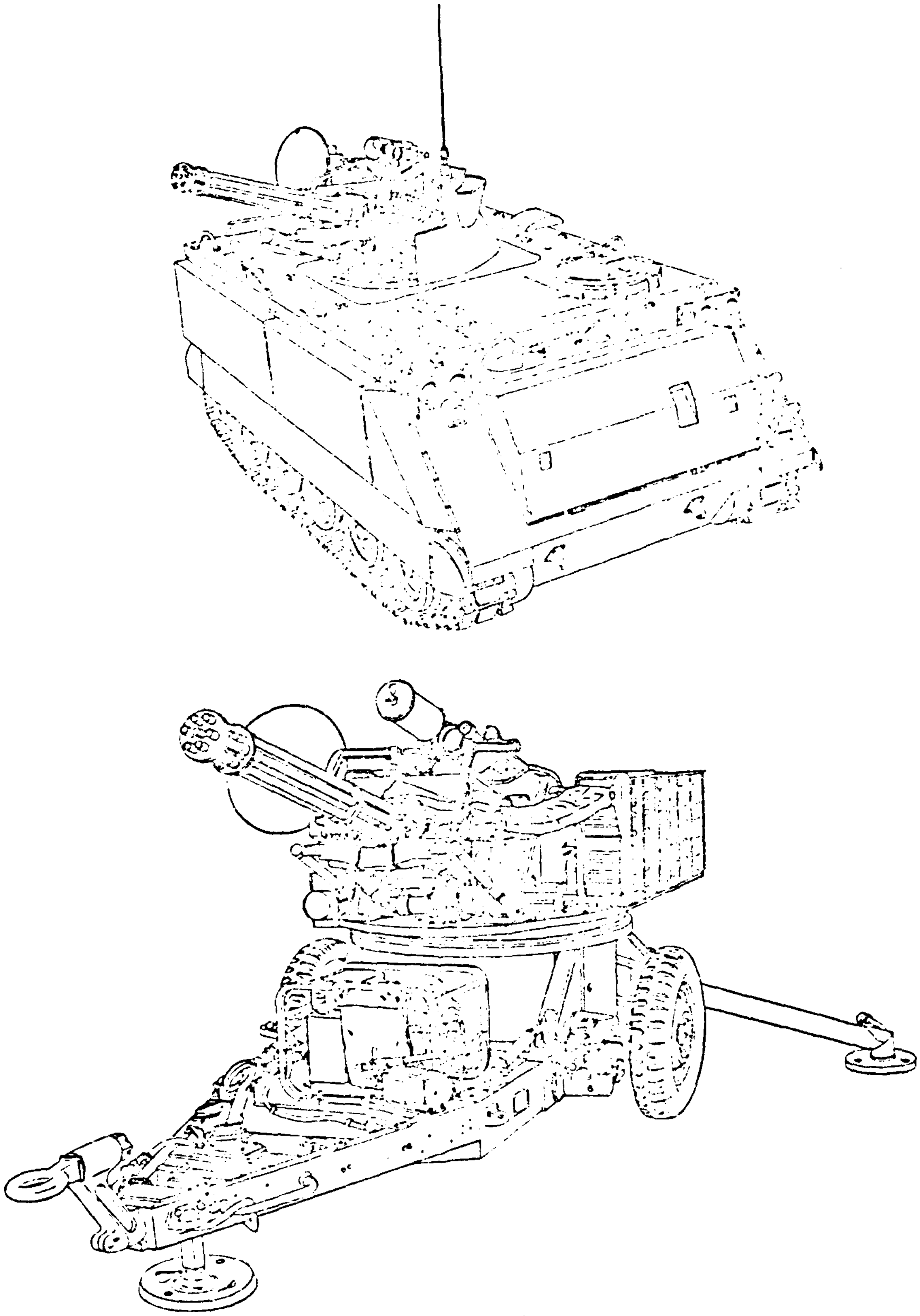


Figure 2.3
Vulcan Gun System (Towed & Self-Propelled)

fielded in the self-propelled configuration only; however, the launching station is a complete, self-contained weapon system, and may be separated from the carrier and operated in a ground-emplaced mode. Chaparral is considered to be a fair weather system capable of operating only during periods of good visibility. The system is composed of three major elements: the launching station, carrier, and Chaparral missiles.

2.2.2 Vulcan:

The Vulcan weapon system comes in two configurations. The M163 is a full-tracked, lightly armored gun system designed for deployment in the forward combat area to provide air defense against the low-altitude air threat. This self-propelled system is capable of delivering a selected rate of fire (1,000 or 3,000 rounds per minute) against air and ground targets with a selectable high rate of fire of 10, 30, 60, or 100 round bursts. The Vulcan towed air defense artillery weapon system consists of a 6-barrel, 20-mm cannon and a fire control system mounted on a 2-wheel trailer carriage. The system is capable of being towed at high speeds over improved roads, travel over rough terrain, and fording shallow streams. Towed Vulcan has basically the same target engagement capability as the self-propelled Vulcan.

2.2.3 Redeye:

Redeye is a short-range, man-portable, shoulder fired air defense guided missile system designed to provide combat units with the capability of destroying low-altitude hostile aircraft. The Redeye weapon consists of three major components: launcher, missile, and launcher battery coolant unit.

2.3 Structure In A Field Environment

The center of the command and control structure in a field environment at any Air Defense level is the command post. The size and location of the command post is primarily determined by the amount of preparation time available for field deployment, the level of command, and the personnel and materiel requirements.

2.3.1 Preparation:

An Air Defense field exercise is usually planned several weeks, even months, in advance. Only in rare instances will a unit be involved in an exercise on a no-notice basis. As a result, most units have sufficient planning and coordination time to deploy to the field with all of their operational resources. This preparation phase typically entails: maintenance of vehicles and weapons, reconnaissance of the exercise area, coordination of communication support, tentative coordination between higher and lower levels of command, and uploading of all equipment needed for field operations. The minimum aount of preparation time necessary for deployment usually increases up the level-of-command chain. A Chaparral or Vulcan Battery needs very little notice (2-4 hours) because of the minimal degree of coordination necessary at that level, while the Group headquarters unit conducts extensive coordination with a variety of military and civilian organizations during its consideration of the entire field operation area.

2.3.2 Command Post:

The command post is the location of the command and control element at each level. Personnel in the command post are responsible for coordinating everything that goes on during the exercise. The size of the command post increases considerably from the Battery up to the Group level. At a Chaparral or Vulcan Air Defense Battery, the command post consists of a small communications center, a small command tent, food and maintenance support, and a small defensive force. At Battalion level, the command post consists of a large communications center, communications support, food and maintenance facilities, and a limited defensive force. The main differences between the Battery and Battalion command posts are the size of the communications center and the large number of people involved in staff functions at the Battalion level. The Air Defense Group has a very large communications center, extensive communications support, food and maintenance facilities, and a defensive force. The physical area for the Group is approximately the same size as the Battalion command post; the primary difference is in the case of the staff sections at the two levels; the Group staff sections are considerably larger--at least one or two officers and two or three enlisted soldiers per section.

The basic command and control facility at Battery and Battalion level is the communications vehicle, an expandable or modified truck containing area maps, status charts, and communications capability between senior and subordinate units. The Group has a mobile communications or command vehicle, which acts as a temporary command post until a more permanent tent site is erected. The tent site includes extensive communications equipment, staff section areas, map overlays, status charts, and a briefing area.

2.3.3 Personnel and Materiel:

For continual operations in a field environment, the command posts at all levels have specific personnel requirements consisting of: an officer-in-charge (OIC), an administration officer or non-commissioned officer (NCO), an intelligence officer or NCO, a plans and operations officer and NCO, a supply officer or NCO, a communications officer or NCO, and several radio and telephone operators. The materiel requirements are equally numerous: communications equipment (secure and non-secure), maps, overlays, status boards, tables, and benches.

The executive officer and the communications and supply sections run the entire command and control operation in the Air Defense Battery. The executive officer assumes the responsibilities of OIC, administration, intelligence, plans, and operations. The communications NCO and his section of radio and telephone operators take care of the communications network, while the supply NCO and his assistant run the supply, logistics, and weapons security tasks. The minimum number of people in the Battery control center is three: the executive officer or his equivalent, a member of the communications section, and a representative from supply. The minimum equipment here is a map of the exercise area showing weapon system positions, and two networks for higher and lower unit coordination.

The Battalion control center has a large contingent in the control cneter. The Battalion executive officer is usually the OIC, with administration, intelligence, supply, and communications officers or NCOs, an operations officer and NCO, and a radio-telephone operator for each of these sections. The minimum number of people needed in the Battalion control center is one for each of the responsibilities listed above. This is justified by having the officer/NCO double as a radio operator. One radio per section, two radio networks for the OIC, maps and unit overlays, and unit status boards are the minimum required equipment for Battalion level operations.

The Group has similar staff section responsibilities as the Battalion, but they are much broader in scope because of the number and different types of subordinate units involved. Each section is at least double the size of a Battalion section, and the equipment and the number of people working in and around the command and control tent is twice the number of the next lower level.

2.4 Operations During A Field Exercise

The personnel and materiel requirements listed in the last section are dictated by the command post operations during a field exercise. Included in these operations are: multiple shifts, a large volume of message traffic, updating charts and overlays, and briefings for individual sections, visitors, and the commander.

2.4.1 Shifts:

The number of shifts per day is usually left up to the section leaders or the OIC to allow flexibility in dealing with the number of people available to handle each job during a 24-hour period. A typical shift at the Battery command and control element includes the OIC, a communications NCO, a supply member, and one or two radio-telephone operators, depending on unit strength. During a shift in the Battalion command and control center, there is an OIC, one representative from the administration, intelligence, supply, and communications sections, an OIC and NCOIC from the operations section, and a minimum of two radio-telephone operators. A shift at Group level is twice as large as the Battalion shift.

2.4.2 Message Traffic:

The size of the shifts is based on the amount of messages coming into and going out of the control center and the need to update the various charts and overlays reflecting unit positions, weapons and ammunition status, personnel strength, missions, and states of alert. Message traffic fluctuates considerably during an exercise, reflecting the time of day, proximity to new missions, enemy activity, and required status reports. Regardless of the situation, there is always scheduled communication between each lower and upper level section with similar responsibilities and between the OICs at each level.

2.4.3 Updating:

A direct result of this message traffic is the constant updating of status charts and map overlays in the command and control center, as preparation for unannounced and scheduled briefings of the current situation. This usually involves aggregation, by section, of all lower level status reports representing unit personnel strength, military intelligence and security information, weapon systems' status, and the weapons' location, logistical, and maintenance status of each unit.

2.4.4 Briefings:

A Battery briefing is usually an informal exchange between the Battery commander and the OIC of the control center to give the commander a complete update on his weapon systems, maintenance activity, personnel, and supply status. A more structured briefing occurs when the commander calls his Platoon leaders into the command post or designated location to issue a new mission statement or update the exercise scenario. Briefings at Battalion level include section members briefing section leaders, section leaders briefing the Battalion commander or special visitors, and the Battalion commander briefing subordinate level commanders and his superior commander. Similar briefings occur at the Group control center on a very regular basis. Some of the more informal briefings may only take a few minutes, while the more formal briefings (staff or command) may last 30 minutes or longer.

CHAPTER 3

MATHEMATICAL PROGRAMMING

Mathematical programming is a decision-maker's tool concerned with optimal, instead of just good, solutions to a problem. Much of the effort in using mathematical programming has dealt with solving applied problems, such as resource allocation. Most of these applied problems are real world problems consisting of numerous levels of complexity; therefore, it is necessary to deal with "subproblems" in order to make them manageable, solvable, and worthwhile for decision-makers at different levels in an organization.

This chapter is intended as an introduction, for the non-mathematically oriented decision-maker, to mathematical programming and how it has been used in the military. An overview of how it can be applied for Air Defense operations in a field exercise environment is also presented.

3.1 Description

Mathematical programming has been used in a field known as management science, which is more commonly called operations research. Operations research is a vague expression, usually encompassing the following characteristics:

1. Deals with the attainment of specified objectives.
2. Considers many alternatives.
3. Tries to select the best alternatives based on the given constraints/critieria (optimization).
4. Deals with an overall systems approach, considering interrelationships of many variables in an environment.

Mathematical programming has become one of the most powerful and flexible tools of operations research available to support decision-making. Bradley, Hax, and Magnanti [5] provide the following definition of mathematical programming:

> It concerns the optimum allocation of limited resources among competing activities, under a set of constraints imposed by the nature of the problem being studied. These constraints could reflect financial, technological, marketing, organizational, or many other considerations. In broad terms, mathematical programming can be defined as a mathematical representation aimed at programming or planning the best possible allocation of scarce resources.

Mathematical programming models, including gaming, simulation, and analytical models, are at the center of very complex decision-making systems in a wide variety of organizations. The problems to which techniques of mathematical programming find optimal solutions are typically constructed as mathematical representations of real world problems. The validity of such models, and the value of the generated solutions, are based on the degree of realism that these models attain. Most of the current methods of finding optimal solutions are basically search methods in which a given solution is continually improved by an interactive procedure until an optimal solution is achieved. When dealing with a particular type of problem, the decision-maker may use an algorithm--a procedure which yields an optimal solution in a finite number of steps--which can be programmed on a computer to solve fairly complex problems in a relatively short period of time. It should be emphasized that mathematical programming models are only representations of the real world and are a supplement to, rather than a substitute for, human judgment in the decision-making process. The best models should be easy to understand

and easy to use.

Mathematical programming is a multifaceted branch of operations research that has a common theoretical base. The primary topics included in mathematical programming are: modeling, linear programming, game theory, networks, integer and mixed integer programming, dynamic and nonlinear programming, inventory theory, and stochastic programming. This study will concentrate on the use of:

1. Analytical modeling, to represent the allocation and scheduling of weapon systems in mathematical terms;
2. Network design, to clearly illustrate the problems facing the decision-makers;
3. Linear programming, to determine the optimal mixture of weapon systems for air defense coverage of assigned objectives;
4. Integer programming, to provide that coverage in the minimum amount of time; and
5. Dynamic programming, to coordinate inputs and outputs from different levels of command.

These techniques will be used as a basis for strategy generation to assist Air Defense commanders in their decision-making process.

Analytical modeling will be used to represent the decision-maker's problem completely in mathematical terms. An objective function, which is a measure of the effectiveness or the value or utility associated with some particular combination of the variables (quantities that are manipulated to achieve some desired outcome [8]) such as

$$Z = f(X_1,\ldots,X_N) \qquad (3.1)$$

will be maximized or minimized, based on a set of mathematical constraints

$$g_i(X_1,\ldots,X_N) \leq b_i, \qquad \text{for } i = 1,\ldots,M. \qquad (3.2)$$

These constraints depict the conditions/restrictions under which the problem is to be decided. This type of model, although not one of the most realistic of models, is one of the least costly and easiest to construct.

A gaming model will also be discussed, as a follow-on result of this study, to show how the air defenders can use the mathematical techniques recommended in this paper to develop an interactive command and control exercise for evaluating the effectiveness of offensive and defensive decision-making.

The Air Defense command and control problem of providing maximum coverage with limited resources in a minimum amount of time can be easily described and presented graphically in network form (Figure 3.1). Once displayed in network form, the problem is more readily analyzed by those involved in the decision-making process. Figure 3.1 illustrates the network of Air Defense weapon system sources (nodes A, B, and C), and the objectives requiring these resources for defense coverage (nodes 1 through N). This command structure has 3 units, denoted by A, B, and C, and N objectives to defend. The number adjacent each source node (TW_j) represents the availability of weapon systems at that node with j used as the unit index. A negative number, W_i, is usually used to indicate a requirement for weapon systems at the i^{th} objective. The number above each arc (links between nodes), $d_{j,i}$, is the distance between the unit and the

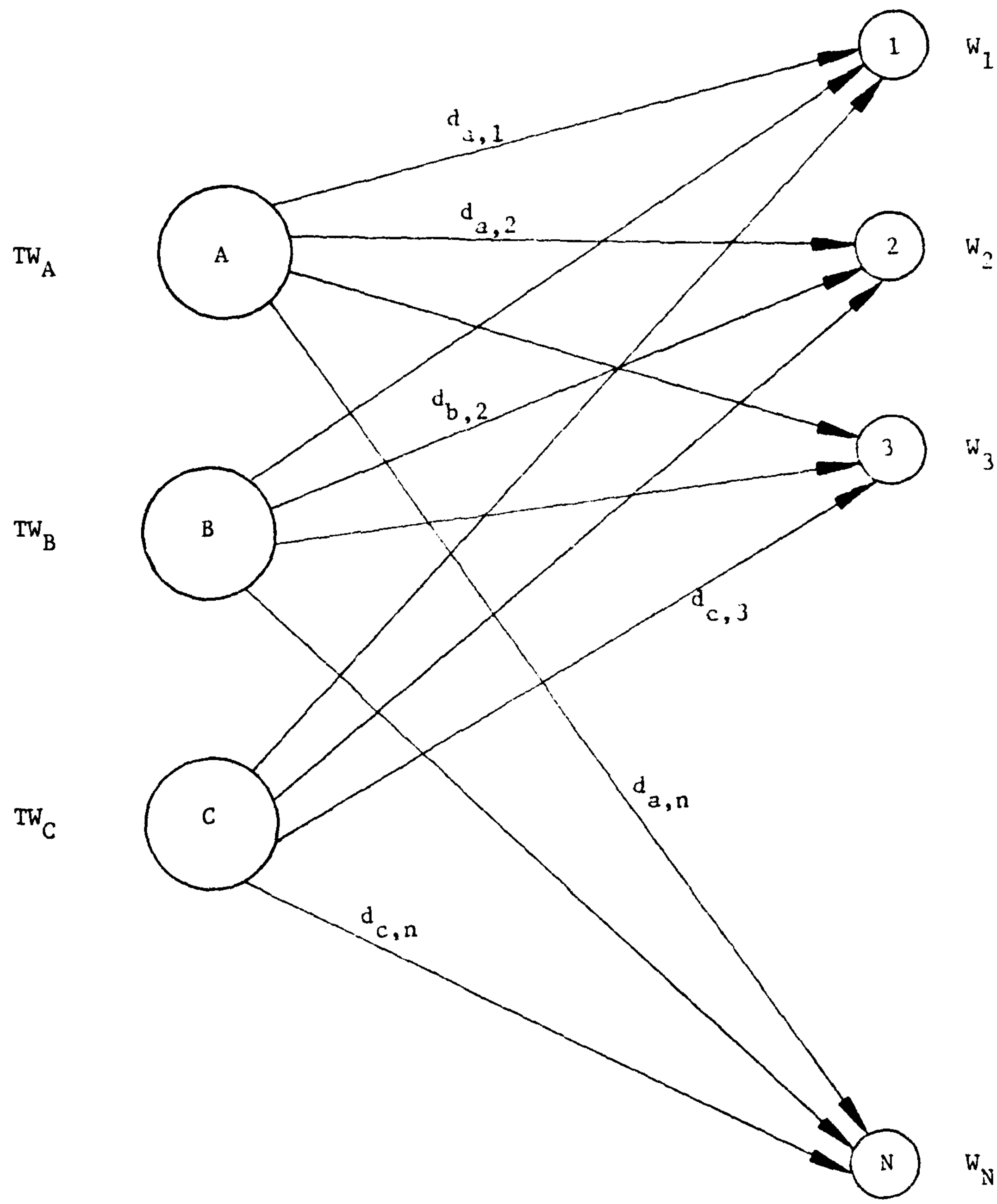


Figure 3.1

General Network Display

objective (source and potential destination). The problem becomes one of determining the best source of each weapon system along an arc so that the total distance/time is minimized and each objective's coverage requirement is satisfied.

The general mathematical programming problem is usually expressed as maximizing the objective function (maximum coverage) represented by Equation (3.1) subject to a variety of constraints, Equation (3.2). The objective function may also be expressed as a minimizing function (minimizing travel time), and the constraints can be expressed in terms of equalities or inequalities. There are N unknowns, $X_1,\ldots,X_N$, and M constraints.

A linear programming problem consists of a linear objective function and constraints. In mathematical notation, it is represented as

$$\text{Maximize } Z = \sum_{j=1}^{N} C_j X_j \qquad (3.3)$$

subject to the following constraints

$$\sum_{j=1}^{N} A_{i,j} X_j \leq B_i, \qquad \text{for } i = 1,\ldots,M \qquad (3.4)$$

including the nonnegativity restrictions,

$$X_j \geq 0, \qquad \text{for } j = 1,\ldots,N. \qquad (3.5)$$

Z	is a scalar
C_j's	are given
X_j's	are the unknowns
$A_{i,j}$'s	are given from a coefficient matrix
B_i's	are given constants--representing acceptable bounds.

For a more detailed explanation of linear programming notation and expanded representation, see [5], [22], or [6].

Integer programming problems are concerned with discrete optimization techniques, most notably the branch-and-bound method and the implicit enumeration method (see [5], [22], or [12]). The notation is similar to the standard linear program, with the addition of an integer value constraint for unknown variables. The additional constraint to equations (3.3), (3.4), and (3.5),

$$X_j \text{ integer}, \qquad j \in (1,2,\ldots,N) \tag{3.6}$$

restricts the decision variables to be integer values necessary when deciding how many weapon systems are required for optimal coverage of the stated objectives. Integer programming provides this method of formulating and solving problems with integrality requirements.

Dynamic programming involves problems that can be divided into "stages" or "phases." At each phase, a decision is required which effects the following phases, in a forward solution approach, or the previous phases in a backwards approach. One or more "states" or "levels" can be identified at each phase which includes specific variables for that level of the decision-making problem.

A definition of dynamic programming presented by McMillian [12] is:

> Dynamic programming is not a general algorithm in the sense of the simplex algorithm of linear programming. Dynamic programming is a kind of approach to solving certain linear and nonlinear programming problems.

3.2 Military Applications

Much of the emphasis and use of mathematical programming in the military has dealt with "upper-echelon" problems, as opposed to day-to-day operation problems of individual military units. Examples of how mathematical programming has been used for military purposes are listed below:

1. Optimize the design of hardened nuclear protective structures to attain a specified level of survivability for minimum cost [16].
2. Design and analysis of an integrated airborne tracking system, airborne threat assessment, and multisensor integration [1].
3. Derivation of mathematical models used in the dynamic flyout section of the Roland missile system simulation program [9].

4. Assessment of the utility of ballistic missile defenses of the silo based US-ICBM force against direct silo attacks [13].
5. Optimize SAM (surface-to-air missile) firing patterns to defend an aircraft carrier [15].
6. Assessment of the performance of area defense weapon systems against air breathing strategic threats [2].
7. Optimize the allocation of tactical missiles between valued targets and defense targets [14].
8. Conduct a study of alternative network architectures for a distributed data processing (DDP) approach to implementation of the BMD site defense data processing subsystem [3].
9. Determine the constraints on excess capability of command and control systems in defense against anti-ship cruise missiles [4].
10. Determine the effectiveness of a three-layer defense against an optimally allocated offense [18].

Most of the attention given to mathematical programming in the military, based on the collection of technical reports in the Defense Technical Information Center (DTIC), deals with strategic analysis of defense systems. This emphasis on strategy evaluation provides decision-makers, primarily involved with operations research and systems analysis projects, with a better understanding of the consequences of their decisions. Very few problems involved with strategy generation and small unit operations have used the modern techniques of mathematical programming. One of the few applications concerns the Marine Aviation Requirements Mathematical Programming Model (MARMAP), which was developed as an analysis tool to provide quantitative support for Marine aviation planners in their day-to-day aviation planning process [10]. Another study involving air defense weapons coverage was conducted at the Huntsville Research Institute under the direction of the Army Missile Command [7]. It dealt with the general category of nonlinear, zero-one programming problems. The emphasis in the present study is on reducing the level of complexity in generating Air Defense coverage guidance, by including weighted objective functions along with time, distance, and weapon system status constraints. The result is a system that can be applied at different levels of Air Defense command and control during a field exercise.

3.3 Air Defense Application

The following overview describes how the techniques of analytical modeling, network design, linear and integer programming, and dynamic programming, can be applied for Air Defense operations at the lowest levels of command and control. A more detailed example of these techniques, using appropriate mathematical representation, is given in the next section.

To avoid the congestion and confusion in the command center at Group, Battalion, and Battery level during an exercise, dispense with the large charts, overlays, and status boards. Avoid the tangle of communication wires, cables, and radio-telephone operators trying to update mission times, weapon systems, and personnel and logistics status. These can be avoided by employing portable computers interacting with each of these levels of command, using an interactive software system. This system could include the Linear Interactive Discrete Optimizer (LINDO) [20] and

[21]), for solving mathematical programming problems, MINITAB [19], for statistical reports, charts, and graphs, and a graphical capability for network displays and map overlays.

A computer programming package, such as LINDO, provides the commander a mathematical procedure for determining the optimal allocation of his scarce resources, the Air Defense weapon system. The combination of a good mathematical formulation method, realistic objectives, and accurate upper and lower bound constraints results in the best allocation of weapon systems to defend the objectives in the shortest amount of time. The optimal coverage strategy can be determined from a set covering problem using a minimum distance matrix $[d(i,j)]$ to generate a time matrix $[t(i,j)]$ and a coverage matrix $[c(i,j)]$ by establishing an upper bound on $t(i,j)$ and setting

$$c(i,j) = \begin{cases} 1 \text{ if } t(i,j) \leq \text{upper bound} \\ 0 \text{ otherwise} \end{cases} \tag{3.7}$$

The optimal coverage strategy is then used by the commander to task organize (structure) his subordinate units into a distinct command organization (i.e., a Battalion), and efficiently assign the stated objectives to his subordinate units. This will involve integer programming to avoid fractional allocation of resources, and matrix generation to determine minimum distances for fastest travel time. The coverage problem can be illustrated by a network model and displayed on a computer terminal view screen by using computer software equipped with a graphical representation of the exercise area. It can be incorporated into the decision-making process as illustrated in the next chapter.

CHAPTER 4

AIR DEFENSE OPERATIONS USING MATHEMATICAL PROGRAMMING

The process of command and control using techniques of mathematical programming during an Air Defense field exercise involves several interrelated phases. In the first phase, two-way communication between parent (Group) and subordinate (Battalion) command centers identifies the objectives to be defended and establishes upper and lower bounds for coverage requirements. In the second phase, the Group command center uses a Linear Programming model with objectives prioritized in the objective function to specify overall coverage requirements satisfying the upper and lower bounds from Phase 1. During Phase 3, the Group command center uses a distance matrix, and converts this to a time matrix representing the amount of time weapon systems from each of the supply locations will take to reach each of the objectives. The decision-maker uses this information along with his knowledge of the mission times, to establish potential coverage assignments. In Phase 4, the Group commander uses inputs from the second and third phase to solve a shortest-distance transportation problem. From this solution, the commander assigns objectives to his Battalions and task organizes his subordinate units into separate commands according to coverage requirements, individual unit status, present unit location, and his own discretion. In the final phase, the Battalion command centers use a transportation model to minimize the distance between unit and objective while still meeting the coverage requirements of the assigned objectives. The Battalion commander then task organizes his subordinate units into separate commands and assigns objectives to his Batteries, using the optimal solution generated by the transportation program after any appropriate modifications of the inputs or the constraints. The phases of this decision-making approach are illustrated in Figure 4.1.

This example will focus on two command levels, the Group and the Chaparral/Vulcan Battalion, in a deterministic setting. Assumptions concerning the exercise have been listed in Section 4.7 and are intended to be representative of actual circumstances involved in such an Air Defense operation. Typically, as the field exercise progresses, more complex decisions involving multiple criteria make the command and control problem much more dynamic. Extensions of this static example will address this complexity and conclude this section.

4.1 Phase 1

The exercise commences when the Group commander receives his initial deployment notification and is given a list of prioritized objectives to defend from enemy attack with his Air Defense assets. The Group command center notifies the Battalions by issuing an early warning order and requests input on upper and lower coverage bounds, using Chaparral and Vulcan weapon systems for each objective. The Battalion command centers use: existing defense plans that include the specified objectives, latest military intelligence sources, input from Battery commanders, if appropriate, and/or a map reconnaissance to provide those coverage bounds. The Group command center consolidates the information from the Battalions and uses it, along with a predesignated priority scheme, to construct a network description of the problem, as illustrated in Figure 4.2.

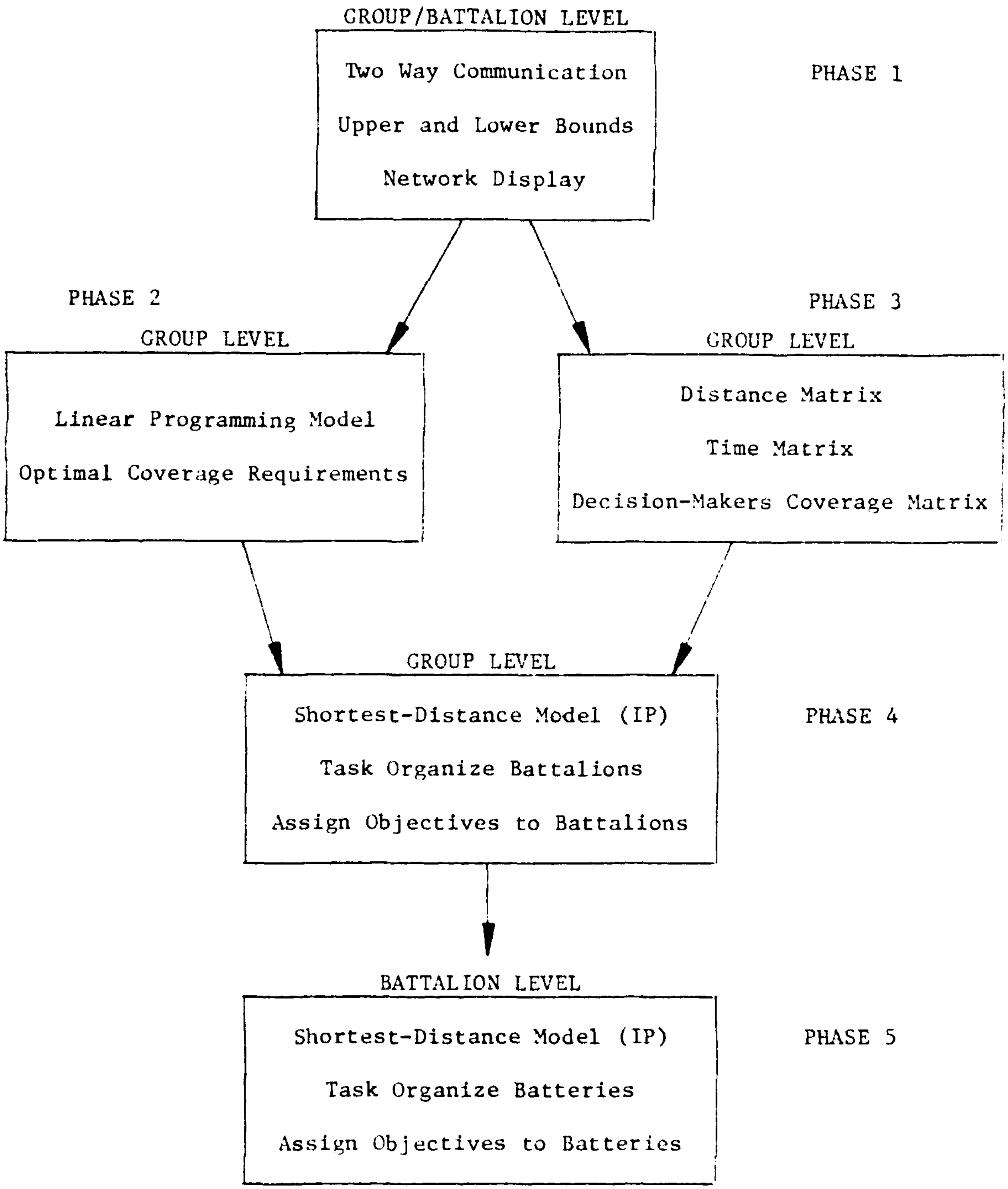


Figure 4.1

Phases of Decision-Making Approach

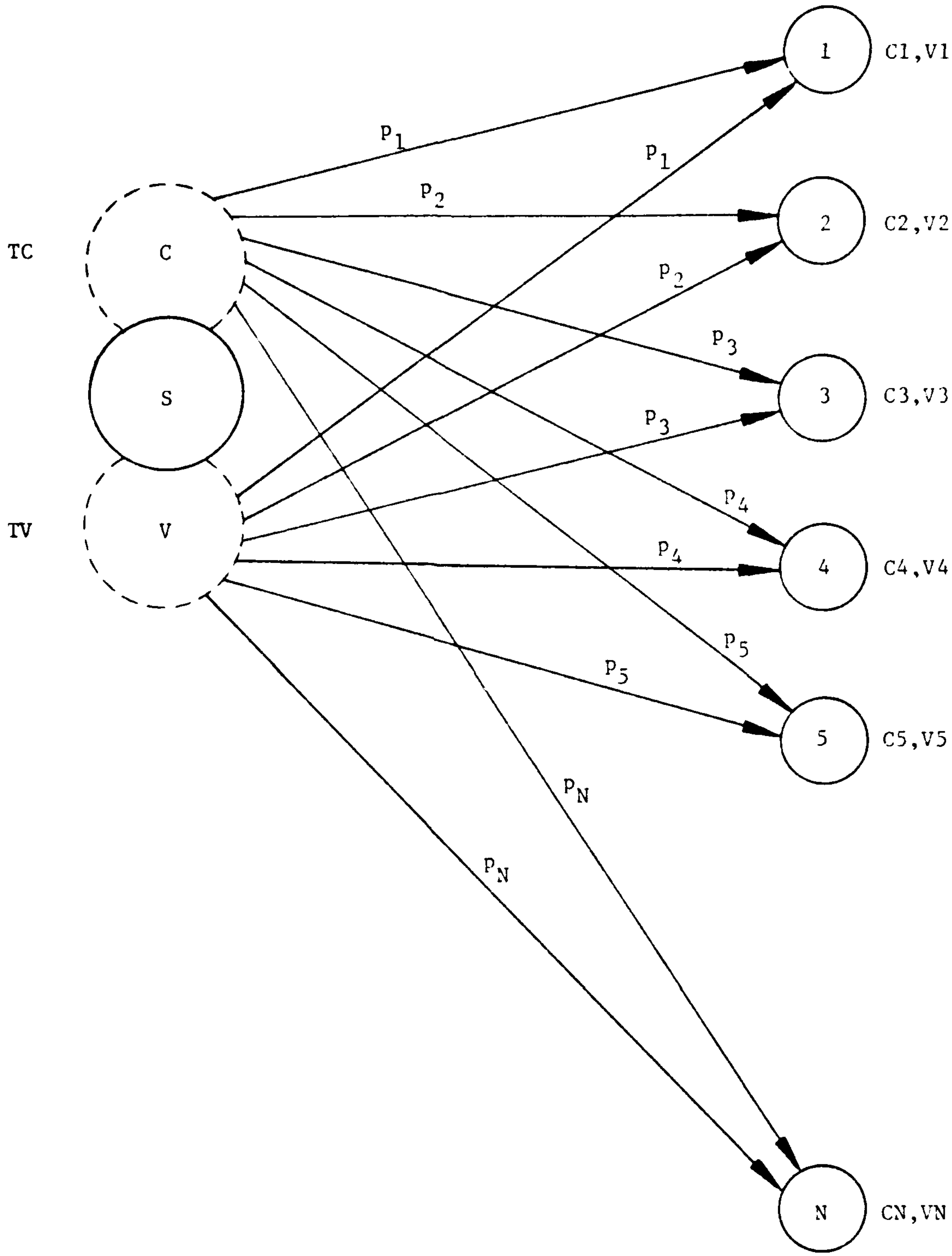


Figure 4.2

Coverage Problem at Group Level in Network Form

4.2 Phase 2

Figure 4.2 depicts the overall coverage problem in network form at the Group level. The supply node, S, represents the Group and its cumulative weapons status. This node actually represents a multi-commodity supply which can be modified by node-splitting. In this case there would be a supply node for each weapon system (C,V), as indicated by the dotted circles. The demand nodes are the objectives and are listed with the possible number of weapon systems appropriate for defensive purposes (C_i, V_i). The arcs joining the supply or source node with the demand or destination nodes represent a weighting system to reflect the relative priority of one objective to another. In this case, objective A1 would be considered a higher priority than A2 if p1 is greater than p2. During Phase 2, the general coverage problem is defined using the following notation:

N = the total number of objectives to be defended,

i = a particular objective from 1 to N,

C_i = the number of Chaparrals assigned to the i^{th} objective,

V_i = the number of Vulcans assigned to the i^{th} objective,

SC_i = the Chaparral slack variable establishing the lower Chaparral coverage bound of the i^{th} objective,

SV_i = the Vulcan slack variable establishing the lower Vulcan coverage bound of the i^{th} objective,

UC_i = the upper coverage bound on the numboer of Chaparral systems for objective i,

UV_i = the upper coverage bound on the number of Vulcan systems for objective i,

USC_i = the lower coverage bound on the number of Chaparrals for objective i established by setting an upper bound on the slack variable,

USV_i = the lower coverage bound on the number of Vulcans for objective i established by setting an upper bound on the slack variable,

TC = the total number of Chaparral weapon systems available,

TV = the total number of Vulcan weapon systems available,

p_i = the number assigned to the i^{th} objective to establish the priority scale.

The linear programming model, in mathematical terms, is:

Maximize $$\sum_{i=1}^{N} p_i C_i + \sum_{i=1}^{N} p_i V_i \tag{4.1}$$

subject to the following constraints:

$$C_i + SC_i = UC_i \quad \text{for } i = 1 \text{ to } N \tag{4.2}$$

$$SC_i \leq USC_i \quad \text{for } i = 1 \text{ to } N \tag{4.3}$$

$$V_i + SV_i = UV_i \quad \text{for } i = 1 \text{ to } N \tag{4.4}$$

$$SV_i \leq USV_i \quad \text{for } i = 1 \text{ to } N \tag{4.5}$$

$$\sum_{i=1}^{N} \leq TC \tag{4.6}$$

$$\sum_{i=1}^{N} \leq TV \tag{4.7}$$

$$C_i, V_i, SC_i, SV_i, UC_i, USC_i, UV_i, USV_i, TC, TV \geq 0 \qquad (4.8)$$

Equation (4.1) is the objective function or goal of the decision-maker, in this case, the Group commander, to maximize the coverage of the objectives (i) according to the priority number (p_i) by optimally using his Chaparral (C_i) and Vulcan (V_i) resources. When expanded, using arbitrary weighting numbers (100,70,90,40,80) for five objectives, equation (4.1) is written as:

$$\begin{aligned} \text{Max} \quad & 100\ C1 + 70\ C2 + 90\ C3 + 40\ C4 + 80\ C5 + \\ & 100\ V1 + 70\ V2 + 90\ Vc + 40\ V4 + 80\ V5 \end{aligned} \qquad (4.1a)$$

Equations (4.2) and (4.3) represent Chaparral upper and lower bound constraints, from Phase 1, for each of the objectives. C_i is the actual number of Chaparrals that will be assigned to cover objective i. UC_i is the number of Chaparrals needed for maximum coverage of objective i, and SC_i is the slack or difference between UC_i and C_i. Establishing lower coverage bounds can be accomplished by setting an upper bound on the Chaparral slack variable, SC_i, as in Equation (4.3) or by setting a lower limit (LC_i) directly on the number of Chaparral weapon systems:

$$C_i \geq LC_i \qquad (4.3a)$$

Equations (4.4) and (4.5) are similar constraints to Equations (4.2) and (4.3) except they deal with the Vulcan weapon system. Again, Equation (4.5) can be rewritten as:

$$V_i \geq LV_i \qquad (4.5a)$$

where LV_i is the lower Vulcan coverage bound for objective i.

Equations (4.6) and (4.7) are the Group's supply constraint for Chaparral and Vulcan weapon systems. The total number of systems allocated for all N objectives has to be less than or equal to the present Group strength of operational Chaparrals (TC) and Vulcans (TV). For the Chaparral weapon system, using 66 as present Group strength, Equation (4.6) is expanded and written as:

$$C1 + C2 + C3 + \ldots + CN \leq 66 \qquad (4.6a)$$

For the decision-maker's convenience and easier sensitivity analysis, it may be more useful to add a slack variable, SC, to this equation. This will allow the Group commander to readily determine how many operational systems are not initially needed. The modified equation is written as

$$C1 + C2 + C3 + \ldots + CN + SC = 66 \qquad (4.6b)$$

The same modification for Equation (4.7) can be made, using SV as the slack variable for excess Vulcans.

Equation (4.8) is a nonnegativity constraint which requires that all variables assume only a positive or zero value. Most LP computer programs assume that all variables are constrained to be nonnegative, so constraints such as Equation (4.8) are unnecessary.

The network representation for this phase using appropriate numbers is illustrated in Figure 4.3 with the complete Linear Programming portion displayed in Appendix 1.

4.3 Phase 3

During Phase 3, distance, time, and potential coverage matrices are generated to insure appropriate units are designated as potential suppliers during the last two phases of the exercise based on mission times and the commander's discretion.

A distance matrix (Table 4.1) consisting of road distances from each Battalion to each objective is used to generate a time matrix as a rough estimate of how long it will take (hours) for personnel to be notified, recalled if necessary, and weapon systems deployed to and set-up at the designated objective. A simple equation taking the inverse of weapon system speed multiplied by the road distance [d(i,j)] plus allotted time for notification and weapon system set-up, results in the matrix entries [t(i,j)] as illustrated in Table 4.2.

The commander, with knowledge of the overall time constraints, can quickly assess the feasible alternatives for each objective from the minimum time matrix [t(i,j)] and immediately translate this into the coverage matrix, [c(i,j)], by setting each matrix element, c(i,j), to zero or one as explained by Equation (3.7).

The coverage matrix is shown in Table 4.3, with an upper bound equal to 6 hours. The Group commander has determined that Battalion A is a potential supply point for objective (demand point) 4, but Battalion B is not a supply point for objective 7.

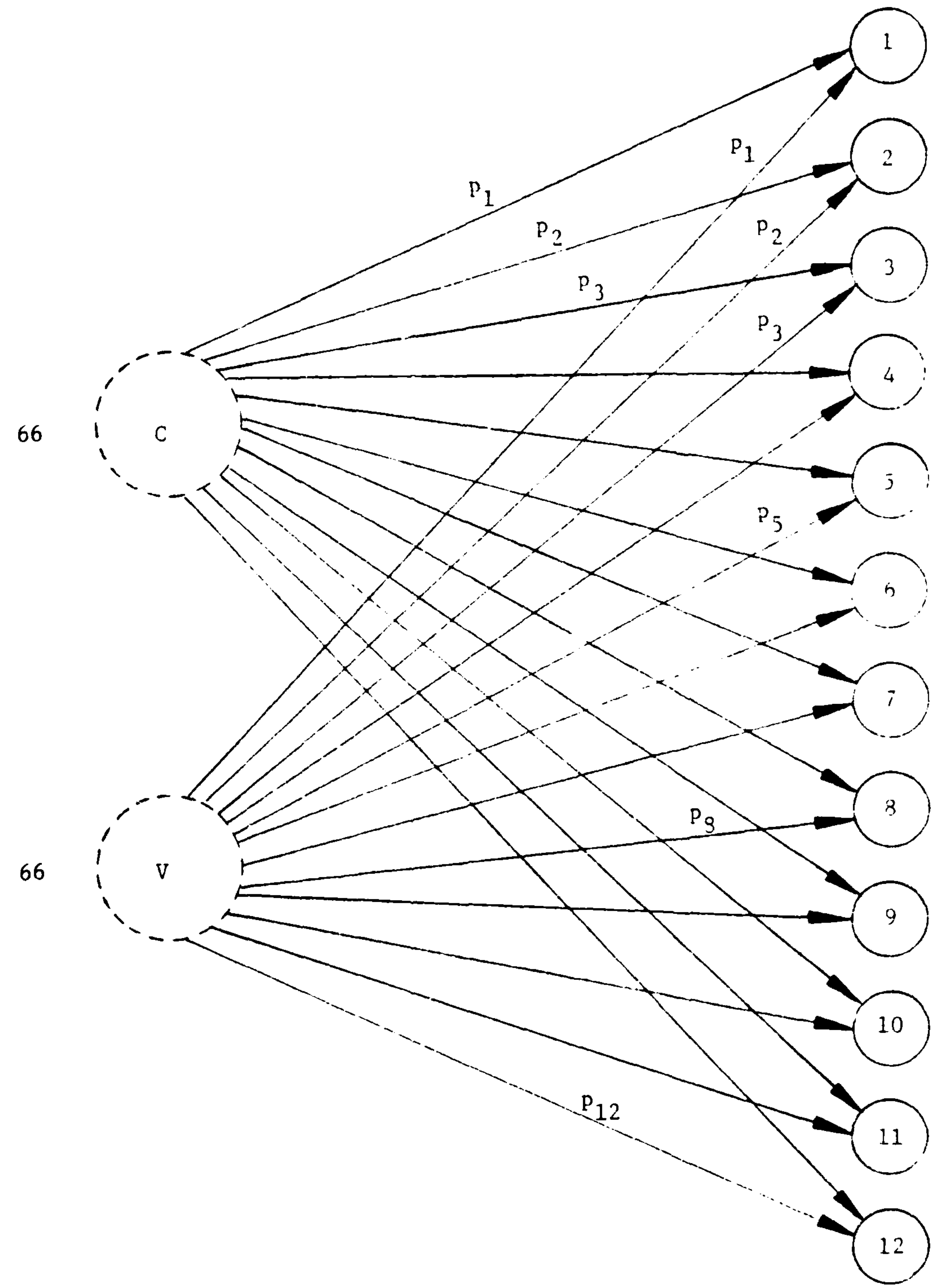


Figure 4.3

Phase 2 in Network Form

Table 4.1

Distance Matrix
(miles): d(i,j)

UNIT	OBJECTIVE 1	2	3	4	5	6	7	8	9	10	11	12	A	B	C
A	10	5	15	10	30	35	40	50	45	35	40	20	-	30	40
B	30	25	28	40	50	55	70	80	75	5	10	50	30	-	70
C	40	45	50	25	60	65	5	10	20	70	75	50	40	70	-

Table 4.2

Time Matrix
(hours: t(i,j)

UNIT	OBJECTIVE 1	2	3	4	5	6	7	8	9	10	11	12
A	2.5	2	3	2.5	4	4.5	5	5.5	5	4.5	5	3
B	4	3.5	3.5	5	5.5	6	6.5	7	6.5	2	2.5	5.5
C	5	5	5.5	3.5	6	6.5	2	2.5	3	6.5	6.5	5.5

Table 4.3

Coverage Matrix
c(i,j)

UNIT	OBJECTIVE 1	2	3	4	5	6	7	8	9	10	11	12
A	1	1	1	1	1	1	1	1	1	1	1	1
B	1	1	1	1	1	1	0	0	1	1	1	1
C	1	1	1	1	1	0	1	1	0	0	1	1

4.4 Phase 4

During this phase, the Group commander uses the information from the previous phases to establish coverage requirements for each objective and construct a shortest-distance mathematical model to minimize travel time and task organize his subordinate Battalions, if appropriate. The overall transportation problem, in network form, is described in Figure 4.4.

Using three Chaparral/Vulcan Battalions (A,B,C), it is most convenient to use the technique of splitting the three supply nodes into six nodes, three supplying Vulcan systems (AV,BV,CV) and three supplying Chaparral systems (AC,BC,CC). The demand nodes have similarly been divided into a Chaparral demand node objective, C_i, and a Vulcan demand node objective, V_i, for each of the N objectives ($i = 1,2,\ldots,N$). Notice that in addition to the arcs connecting supply and demand nodes, there are also arcs connecting each of the distinct weapon system supply nodes. This allows the transfer of weapon systems between Battalions, maintaining Battery integrity, which is the basis of task organizing each Battalion. Maintaining Battery integrity allows the transfer of one Battery (12 weapon systems) from one Battalion to the other.

The notation involved in the integer programming problem is:

i	=	designated objective from 1 to N,
j,k,J, and K	=	designations for Battalions A,B,C,
JC_i	=	a 0-1 variable determining allocations of Chaparrals from Battalion J to objective i; $J \in \{A,B,C\}$,
JV_i	=	a 0-1 variable determining allocations of Vulcans from Battalion J to objective i; $J \in \{A,B,C\}$,

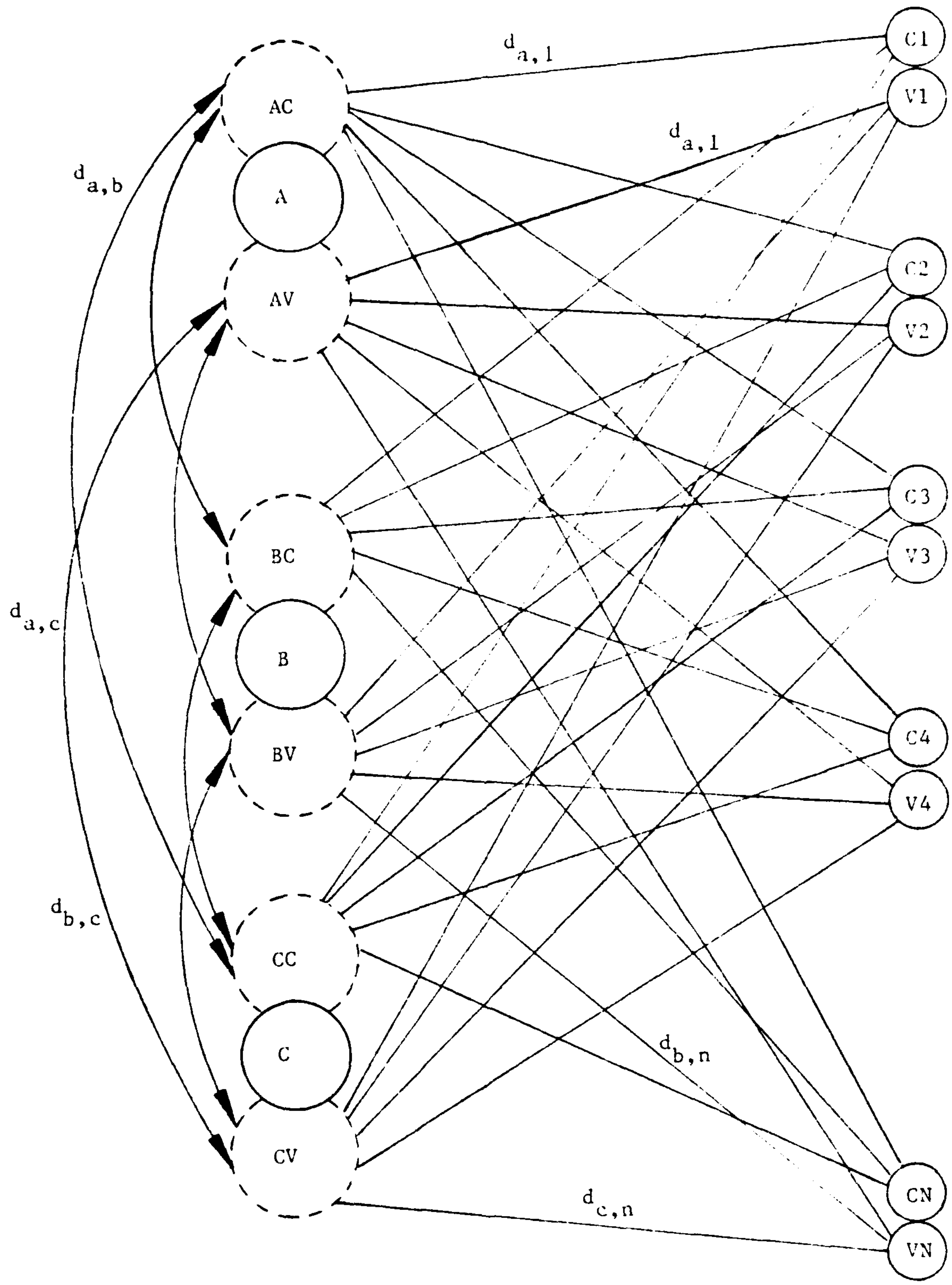


Figure 4.4

Phase 4 Weapon System Transportation Network

CJK = a 0-1 variable for Chaparrals transferred from Battalion J to Battalion K. $J \varepsilon \{A,B,C\}$; $K \varepsilon \{A,B,C\}$; and $J \neq K$,

VJK = a 0-1 variable for Vulcans transferred from Battalion J to Battalion K. $J \varepsilon \{A,B,C\}$; $K \varepsilon \{A,B,C\}$; and $J \neq K$,

$d_{j,i}$ = distance from Battalion j to objective i,

$d_{j,k}$ = distance from Battalion j to Battalion k; $j \neq k$,

JC = the total Chaparrals assigned to the objectives from Battalion J,

JV = the total Vulcans assigned to the objectives from Battalion J,

C_i = the optimal number of Chaparral systems covering objective i determined from Phase 3,

V_i = the optimal number of Vulcan systems covering objective i determined from Phase 3,

SJC = a Chaparral slack variable; the number of Chaparrals unused at Battalion J,

SJV = a Vulcan slack variable; the number of Vulcans unused at Battalion J,

c(i,j) = a 0-1 variable from the coverage matrix of Phase 3.

The mathematical model is written as:

$$\text{Minimize} \quad \sum_{i=1}^{N} d_{a,i} AC_i + \sum_{i=1}^{N} d_{a,i} AV_i$$

$$+ \sum_{i=1}^{N} d_{b,i} BC_i + \sum_{i=1}^{N} d_{b,i} BV_i \qquad (4.9)$$

(continued)

$$+ \sum_{i=1}^{N} d_{c,i} CC_i + \sum_{i=1}^{N} d_{c,i} CV_i$$

$$+ \quad d_{a,b}(CBA + CAB + VBA + VAB)$$

$$+ \quad d_{a,c}(CCA + CAC + VCA + VAC) \tag{4.9}$$

$$+ \quad d_{b,c}(CBC + CCB + VBC + VCB)$$

subject to the following constraints:

$$AC = \sum_{i=1}^{N} C_i AC_i \tag{4.10}$$

$$AV = \sum_{i=1}^{N} V_i AV_i \tag{4.11}$$

$$BC = \sum_{i=1}^{N} C_i BC_i \tag{4.12}$$

$$BV = \sum_{i=1}^{N} V_i BV_i \tag{4.13}$$

$$CC = \sum_{i=1}^{N} C_i CC_i \tag{4.14}$$

$$CV = \sum_{i=1}^{N} V_i CV_i \tag{4.15}$$

$$AC - 12\ CBA - 12\ CCA + 12\ CAB + 12\ CAC + SAC = 24 \tag{4.16}$$

$$CAB + CAC \leq 1 \tag{4.17}$$

$$CBA + CCA \leq 1 \tag{4.18}$$

$$AV - 12\ VBA - 12\ VCA + 12\ VAB + 12\ VAC + SAV = 24 \quad (4.19)$$

$$VAB + VAC \leq 1 \quad (4.20)$$

$$VBA + VCA \leq 1 \quad (4.21)$$

$$BC - 12\ CAB - 12\ CCB + 12\ CBA + 12\ CBC + SBC = 24 \quad (4.22)$$

$$CBA + CBC \leq 1 \quad (4.23)$$

$$CAB + CCB \leq 1 \quad (4.24)$$

$$BV - 12\ VAB - 12\ VCB + 12\ VBA + 12\ VBC + SBV = 24 \quad (4.25)$$

$$VBA + VBC \leq 1 \quad (4.26)$$

$$VAB + VCB \leq 1 \quad (4.27)$$

$$CC - 12\ CAC - 12\ CBC + 12\ CCA + 12\ CCB + SCC = 24 \quad (4.28)$$

$$CCA + CCB \leq 1 \quad (4.29)$$

$$CAC + CCB \leq 1 \quad (4.30$$

$$CV - 12\ VAC - 12\ VBC + 12\ VCA + 12\ VCB + SCV = 24 \quad (4.31)$$

$$VCA + VCB \leq 1 \quad (4.32)$$

$$VAC + VBC \leq 1 \quad (4.33)$$

$$AC_i + BC_i + CC_i = 1 \qquad \text{for } i = 1 \text{ to } N \qquad (4.34)$$

$$AV_i + BV_i + CV_i = 1 \qquad \text{for } i = 1 \text{ to } N \qquad (4.35)$$

$$AC_i - AV_i = 0 \qquad \text{for } i = 1 \text{ to } N \qquad (4.36)$$

$$BC_i - BV_i = 0 \qquad \text{for } i = 1 \text{ to } N \qquad (4.37)$$

$$CC_i - CV_i = 0 \qquad \text{for } i = 1 \text{ to } N \qquad (4.38)$$

$$AC_i = c(i,a) \qquad \text{for } i = 1 \text{ to } N \qquad (4.39)$$

$$BC_i = c(i,b) \qquad \text{for } i = 1 \text{ to } N \qquad (4.40)$$

$$CC_i = c(i,c) \qquad \text{for } i = 1 \text{ to } N \qquad (4.41)$$

$$AC,AV,BC,BV,CC,CV,SAC,SAV,SBC,SBV,SCC,SCV,C_i,V_i \geq 0 \qquad (4.42)$$

$$\begin{gathered} AC_i,AV_i,BC_i,BV_i,CC_i,CV_i,CBA,CCA,CAB,CAC, \\ VBA,BCA,VAB,VAC,CCB,CBC,VCB,VBC,c(i,j) = \text{integer } 0 \text{ or } 1 \end{gathered} \qquad (4.43)$$

Equation (4.9) is the objective function which minimizes the distance between the three Battalions and the assigned objectives. The coefficients, $d_{a,i}$, $d_{b,i}$, and $d_{c,i}$, are the arc lengths between supply and demand nodes, while $d_{a,b}$, $d_{a,c}$, and $d_{b,c}$ are the transfer distances between supply nodes, which allows for rearranging each Battalion's command structure. These distances are represented by arcs in the network display (Figure 4.4), and can be read directly from the distance matrix in Table 4.1. AC_i, BC_i, CC_i, AV_i, BV_i, and CV_i are the 0-1 variables such that

$$AC_i = \begin{cases} 1 \text{ if Chaparrals from Battalion A defend} \\ \quad \text{or supply objective i;} \\ 0 \text{ otherwise.} \end{cases} \qquad (4.9a)$$

$$AV_i = \begin{cases} 1 \text{ if Vulcans from Battalion A defend or} \\ \quad \text{supply objective i;} \\ 0 \text{ otherwise.} \end{cases} \qquad (4.9b)$$

CBA, CAB, VBA, VAB,...,VCB are the 0-1 variables representing weapon system transfers between Battalions so that

$$CBA = \begin{cases} 1 \text{ if a Chaparral Battery from Battalion B} \\ \quad \text{is transferred to Battalion A;} \\ 0 \text{ otherwise.} \end{cases} \qquad (4.9c)$$

$$VBA = \begin{cases} 1 \text{ if a Vulcan Battery from Battalion B} \\ \quad \text{is transferred to Battalion A;} \\ 0 \text{ otherwise.} \end{cases} \qquad (4.9d)$$

Using the distance matrix of Table 4.1, one term from Equation (4.9), $\sum_{i=1}^{N} d_{a,i} AC_i$, can be expanded as

$$10\ AC1 + 5\ AC2 + 15\ AC3 + 10\ AC4 + 30\ AC5 + \ldots + 20\ AC12 \qquad (4.9e)$$

Equations (4.10), (4.12), and (4.14) determine how many Chaparrals are allocated from each Battalion with C_i equal to the number of Chaparrals required for the i^{th} objective. This number, C_i, was determined from Phase 2 and can be read directly from the solution printout for Model 2 in Appendix 1.

Equations (4.11), (4.13), and (4.15) determine how many Vulcan systems are allocated from each Battalion, with V_i read directly from the solution printout for Phase 2. Expanding Equations (4.11) and (4.12) results in the constraints

$$AC = 8\ AC1 + 6\ AC2 + 5\ AC3 + \ldots + 4\ AC12 \qquad (4.11a)$$

$$AV = 8\ AV1 + 6\ AV2 + 6\ AV3 + \ldots + 4\ AV12 \qquad (4.12a)$$

Equations (4.16), (4.19), (4.22), (4.25), (4.28), and (4.31) insure that no more weapon systems than are physically present are assigned from each supply node (Battalion). To facilitate sensitivity analysis, each of these equations includes slack variables (SAC, SAV,...,SCV) and strict equalities. These variables indicate how many weapon systems each Battalion has in reserve.

Equations (4.17) and (4.18), (4.20) and (4.21), (4.23) and (4.24), (4.26) and (4.27), (4.29) and (4.30), (4.32) and (4.33) allow for weapon system transfer between Battalions in Battery strength. These equations establish the number of Batteries allowed to be transferred out of or into each Battalion. In this example, a maximum of 1 Chaparral Battery consisting of 12 weapon systems and 1 Vulcan Battery of 12 systems, can be transferred out of a Battalion or into a Battalion. These transfer variables, CAB, CAC,...,VAC, and VBC are the 0-1 variables described by Equations (4.9c) and (4.9d).

Equations (4.34) and (4.35) insure that each objective is covered by Chaparral resources (4.34) from one of the three Battalions and Vulcan resources (4.35) while Equations (4.36), (4.37), and (4.38) guarantee that Battalion integrity is maintained for each objective. This translates into having Chaparral and Vulcan assets defending the i^{th} objective from the same Battalion.

Equations (4.39), (4.40), and (4.41) insure that only potential assignments of Chaparrals to a given objective are considered as specified by the commander in Phase 3 with his coverage matrix (Table 4.3). There is no need to include additional equations for the Vulcan weapon systems because Equations (4.36), (4.37), and (4.38), along with (4.39), (4.40), and (4.41) already take them into account.

Equation (4.42) is the nonnegativity constraint for the variables while Equation (4.43) distinguishes this as a 0-1 Integer Programming problem.

After solving this problem, the Group commander task organizes his Battalions and assigns them the appropriate objectives, unless he wishes to modify the constraints or change selected inputs, based on his own discretion.

4.5 Phase 5

Once the Battalion commander receives his directions from the Group commander concerning the Battalion's configuration and the assigned objectives, he uses a network design and develops a mathematical formulation similar to Phase 4 to minimize the distance to the objectives while still meeting the coverage requirements from Phases 2 and 3. After obtaining his solution and making any desired changes, the Battalion commander task organizes his Batteries and assigns them the appropriate objectives. Using Battalion C with two Chaparral Batteries and two Vulcan Batteries, the network model is illustrated in Figure 4.5. Also incorporated into this level are four Redeye or Stinger weapon systems, which can be used by the Battalion commander to augment a Chaparral unit, and the transfer, at Platoon strength, of Vulcan systems between Vulcan Batteries ($\pm$ 4).

The notation used for the mathematical model of this phase is:

m = the number of objectives assigned to Battalion C,

i = index for the objectives from 1 to M,

a_i = the distance from Battery A to objective i,

b_i = the distance from Battery B to objective i,

c_i = the distance from Battery C to objective i,

d_i = the distance from Battery D to objective i,

$c_d(d_c)$ = the distance from Battery C(D) to Battery D(C),

$r_a(r_b)$ = the distance from Battalion to Battery A(B),

$CCA_i(CCB_i)$ = 0-1 variables representing Chaparral coverage from Battery A(B) of Battalion C for objective i,

$VCC_i(VCD_i)$ = 0-1 variables representing Vulcan coverage from Battery C(D) of Battalion C for objective i,

CCA(CCB) = the total number of Chaparral systems supplied from Battery A(B),

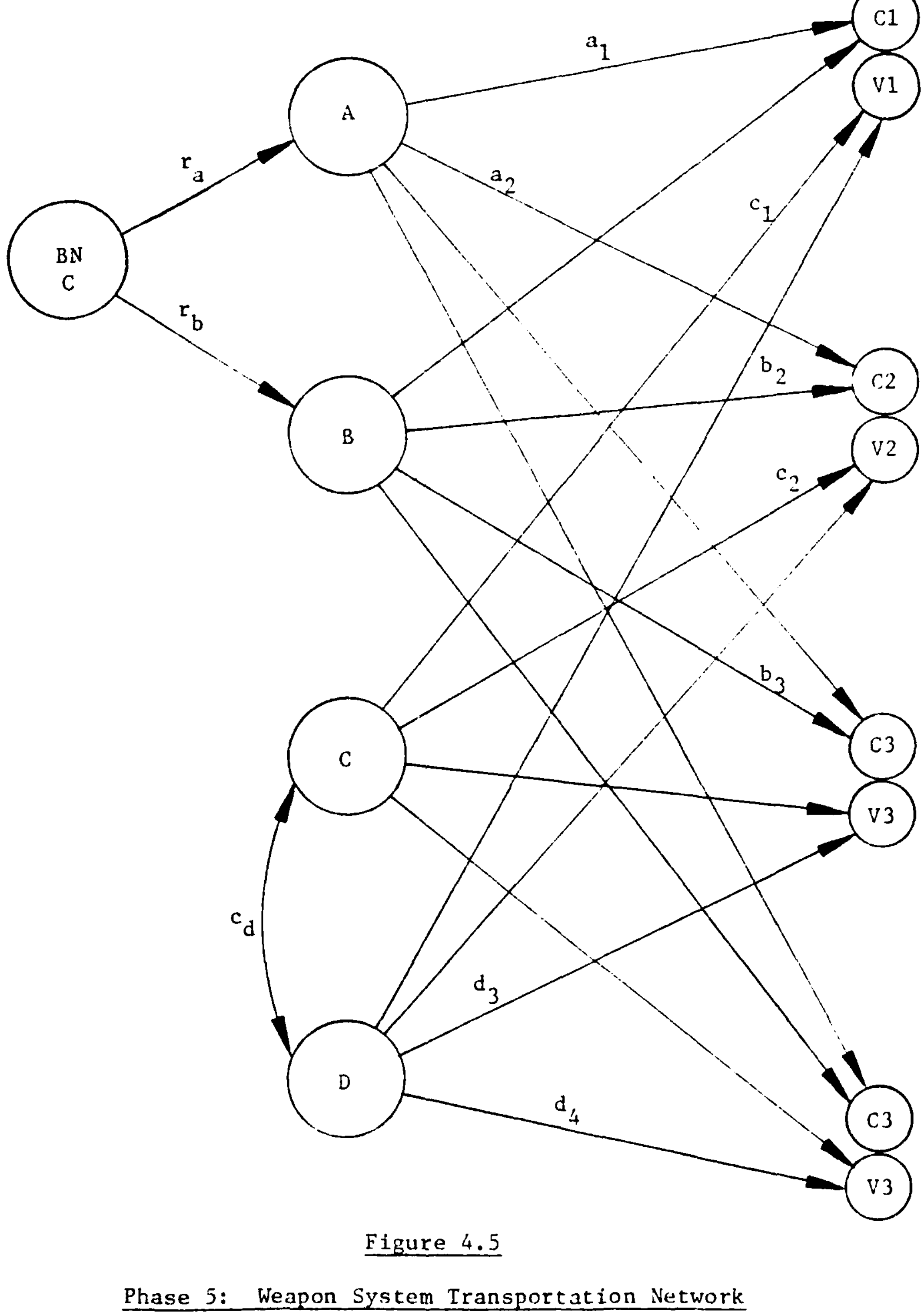


Figure 4.5

Phase 5: Weapon System Transportation Network

VCC(VCD) = the total number of Vulcan systems supplied from Battery C(D),

SCCA(SCCB) = the Chaparral slack variable; the unused number of Chaparrals at Battery A(B),

SVCC(SVCD) = the Vulcan slack variable; the unused number of Chaparrals at Battery C(D),

RA(RB) = 0-1 variables for the assignment of Redeye systems from the Battalion to Battery A(B),

VCCD(VCDC) = 0-1 variables allowing transfer of Vulcan systems from Battery C(D) to Battery D(C).

The integer programming model is formulated as:

$$\text{Minimize} \quad \sum_{i=1}^{M} a_i CCA_i + \sum_{i=1}^{M} b_i CCB_i + \sum_{i=1}^{M} c_i VCC_i + \sum_{i=1}^{M} d_i VCD_i + r_a RA + r_b RB + d_c VCDC + c_d VCCD \tag{4.44}$$

subject to the following constraints:

$$CCA = \sum_{i=1}^{M} C_i CCA_i \tag{4.45}$$

$$CCB = \sum_{i=1}^{M} C_i CCB_i \tag{4.46}$$

$$VCC = \sum_{i=1}^{M} V_i VCC_i \tag{4.47}$$

$$VCD = \sum_{i=1}^{M} V_i VCD_i \tag{4.48}$$

$$CCA - SCCA - 4RA = 10 \tag{4.49}$$

$$CCB - SCCB - 4RB = 10 \tag{4.50}$$

$$VCC - SVCC + 4VCCD - 4VCDC = 12 \tag{4.51}$$

$$VCD - SVCD + 4VCDC - 4VCCD = 12 \tag{4.52}$$

$$RA + RB \leq 1 \tag{4.53}$$

$$\sum_{i=1}^{M} (CCA_i + CCB_i) = 4 \tag{4.54}$$

$$\sum_{i=1}^{M} (VCC_i + VCD_i) = 4 \tag{4.55}$$

$$CCA, CCB, VCC, VCD, SCCA, SCCB, SVCC, SVCD, C_i, V_i \geq 0 \tag{4.56}$$

$$CCA_i, CCB_i, VCC_i, VCD_i, RA, RB, VCCD, VCDC = \text{Integer 0 or 1} \tag{4.57}$$

Equation (4.44) is the objective function minimizing total distance traveled between the supply nodes (Battery or Battalion) and the objectives or demand nodes (see Table 4.4). CCA_i, CCB_i, VCC_i, and VCD_i are the 0-1 variables so that

$$CCA_i = \begin{cases} 1 \text{ if Chaparrals from Battery A cover objective i,} \\ 0 \text{ otherwise.} \end{cases} \tag{4.44a}$$

$$VCC_i = \begin{cases} 1 \text{ if Vulcans from Battery C cover objective i,} \\ 0 \text{ otherwise.} \end{cases} \tag{4.44b}$$

Table 4.4

Phase 5 Road Distances Between Units and Objectives

UNIT	OBJECTIVE				
	4	7	8	9	3N
A	25	5	10	20	5
B	35	10	15	5	10
C	25	5	10	20	5
D	35	10	15	5	10

RA and RB are 0-1 variables with the following values:

$$RA = \begin{cases} 1 \text{ if the 4 Redeyes are assigned from the} \\ \quad \text{Battalion to Battery A,} \\ 0 \text{ otherwise.} \end{cases} \tag{4.44c}$$

$$RB = \begin{cases} 1 \text{ if the 4 Redeyes are assigned from the} \\ \quad \text{Battalion to Battery B,} \\ 0 \text{ otherwise.} \end{cases} \tag{4.44d}$$

The mathematical formulation can also be adjusted to provide more flexibility in assigning individual Redeye systems, if desired.

VCDC and VCCD are also 0-1 variables representing the transfer of Vulcan systems from one Vulcan Battery to the other.

$$VCDC = \begin{cases} 1 \text{ if one Vulcan Platton (4 weapons) is} \\ \quad \text{transferred from D to C Battery,} \\ 0 \text{ otherwise.} \end{cases} \tag{4.44e}$$

$$VCCD = \begin{cases} 1 \text{ if one Vulcan Platoon is transferred} \\ \quad \text{from C to D Battery,} \\ 0 \text{ otherwise.} \end{cases} \tag{4.44f}$$

Equations (4.45) through (4.48) describe the number of weapon systems, from each Battery, used for the objectives.

Equations (4.49) through (4.52) insure that the supply of weapon systems from each unit is not violated and include necessary information (slack variables) to aid the Battalion commander when he task organizes his subordinate units.

Equation (4.53) uses the 0-1 variables RA and RB to determine only one assignment of the 4 Redeye weapons.

Equations (4.54) and (4.55) guarantee that all the objectives will be covered by both Chaparral and Vulcan weapon systems.

Equation (4.56) is the nonnegativity constraint, and Equation (4.57) imposes integer constraints on the problem.

Solving this integer programming problem gives the Battalion commander the information he needs to accurately organize his Batteries and assign objectives to provide the best possible coverage during the Air Defense field exercise.

The five phases described in this example illustrate the use of mathematical programming techniques including: network theory, linear and integer programming, and dynamic programming applied to a multi-level decision-making problem. Output from one phase provides the input to the next phase until all "subproblems" are addressed.

4.6 Command Post Activities

During the five phases of this decision-making process, there are many activities taking place in the command and control centers which provide inputs to the decision-makers. To complete the mathematical programming approach to Air Defense field operations, statistical and graphical techniques are included into a "total" software computer package which provides the decision-makers with the necessary information on all aspects of the exercise. Combining statistical and graphical capabilities with the mathematical programming model allows immediate updating of network displays and mathematical equations which results in optimal solutions based on accurate information. This will also alleviate the confusion, long preparation time, and space and personnel requirements traditionally experienced during briefings at each level of command and control.

As described in Section 2.3, the command post at Group and Battalion level are divided into the following sections:

Administration and Personnel	(S-1)
Intelligence and Information Security	(S-2)
Plans, Operations, and Communications	(S-3)
Logistics and Maintenance	(S-4)
Staffing the Command Post	(HHB)

Each of these sections is responsible for several status charts or statistical reports that have a direct or indirect effect on the commander's decision-making process when deploying his Air Defense assets. Some of these reports and charts are listed by section below:

S-1
- Personnel Status Report
- Casualty Report
- Incoming Personnel Report

S-2
- Friendly/Enemy Force Situation
- Weather Forecasts
- Prisoner-of-War (POW) Report
- Communications Security Report

S-3
- Weapon System Status Report
- Defensive Coverage and Map Overlays
- After Action Report
- NBC Report
- Signal/Communication Status Report

S-4
- Ammunition Status Report
- Fuel Status Report
- Maintenance Status Report

HHB
- Command Post Overlay
- Security Force Requirements
- Schedule of Events

These charts, overlays, and reports can be maintained in the computer memory and are updated continuously to provide accurate inputs to the mathematical programming formulations and network displays. Examples of two status reports at the Group and Battalion level are illustrated in Figures 4.6 and 4.7. The structure of these reports and charts is relatively constant. The only input needed would be the actual numbers. The same is true for the distance and matrix formats described in Phases 2 and 3. The Defense Mapping Agency cartographers have already programmed and stored maps and overlays in computer software packages [11]. Based on the expected area of operations, the Air Defense command elements would already have these on hand.

4.7 Assumptions

The following assumptions were made for the five phase example. They are based on the author's experience in command and control elements in Europe, and can be applied to a majority of Air Defense field exercises.

1. There are sufficient Air Defense resources at each level of command to meet minimum coverage requirements for all assigned objectives.
2. At the start of the exercise, status reports accurately reflect the operational strength of individual units.

BATTALION LEVEL

	AUTHORIZED	ASSIGNED	PRESENT FOR DUTY	% AUTH. STRENGTH
OFFICER				
NCO				
ENLISTED				

BN

GROUP LEVEL

GRADE	AUTHORIZED			ASSIGNED			PRESENT FOR DUTY			% AUTH. STRENGTH		
	A	B	C	A	B	C	A	B	C	A	B	C
OFFICER												
NCO												
ENLISTED												

Figure 4.6

Personnel Status Report

BATTALION/GROUP LEVEL

STATUS

	CHAPARRAL			VULCAN			REDEYE	
BN/BTRY	RFA	M-8hr.	OA	RFA	M-8hr.	OA	RFA	OA
A								
B								
C								
D								

RFA: Ready for Action.

M-8hr: Requires 8 hours maintenance before RFA.

OA: Out of Action--requires more than 8 hours maintenance.

Figure 4.7

Weapon Status Report

3. The Air Defense Battalions are structured with pure Batteries of either Chaparral weapon systems or Vulcan weapon systems.

4. Secure communications links between portable computers at different command centers are established by the Signal Corps personnel.

5. Sufficient early warning times allow for two-way communications between each level of command and control during Phase 1.

6. Mixing of weapon systems from different Battalions to cover an objective is not allowed.

7. No more than one Chaparral and one Vulcan Battery may be transferred in or out of a Battalion.

8. No more than one Vulcan Platoon may be transferred in or out of a Battery.

9. Redeye weapons (4 teams) can be assigned directly to either Chaparral Batteries by the Battalion commander.

10. Distances for Phases 3, 4, and 5 will use the Battalion and Battery command post locations as the origin.

11. Road distances will reflect primary routes of march in terms of miles.

4.8 Extensions

Many of the parameters in the five phase example can be readily expanded upon. The mathematical formulation described in each of the phases can include a larger number of units and objectives, and a greater variety of weapon systems.

The specific structure of the Battery/Battalion/Group is relatively unimportant. The general concept of their command and control interaction is the main concern.

Phase 1 can be modified, as time dictates, to leave the coverage requirements up to the discretion of the Group's plans and operations personnel.

During the example, a fixed number of weapon systems, predesignated priority objectives, and predetermined road distances were used. To make this approach more flexible, a complete software system can be developed which combines linear and integer programming model generator and solution algorithms, statistical reports and charts, and graphical capabilities for network design and map overlays that interact with each other to provide a flexible and responsive decision-making tool. With these capabilities, the command centers can react to changes in weapon system, objective, or distance variables. When new missions are assigned, the appropriate information can be entered and new optimal coverage and transportation solutions can be generated. When weapon systems are destroyed or are otherwise put out of action, analysis of status charts and slack variables can quickly yield alternatives for restructuring the defense or resupplying the appropriate units. Resupplies of weapon systems, ammunition, fuel, and personnel can be directly incorporated into the mathematical model and the augmentation by entire units can be easily evaluated and included into the decision-making problem.

This type of flexibility complements Air Defense command and control centers in both static exercises and more dynamic environments.

CHAPTER 5

ANALYSIS OF A MATHEMATICAL PROGRAMMING APPROACH

The following sections discuss the advantages and disadvantages of applying mathematical programming techniques to current Air Defense operations during a field exercise.

5.1 Advantages

The use of mathematical programming for small unit Air Defense operations helps provide optimal allocation of military resources, reduces the amount of time needed for successful mission accomplishment, and allows a much greater amount of flexibility during the exercise.

Displaying the exercise in a network model not only provides a clear depiction of the weapon's allocation problem, but also leads to an integer covering problem which helps insure the optimal allocation of Air Defense weapon systems with coordinated interaction throughout the multiple levels of command and control. This type of approach reduces the staff requirements in the command post, in terms of personnel and equipment. Assistant staff personnel, both officer and NCO, are no longer needed to perform jobs that can be monitored by one person per section. They can become more involved in evaluating and training subordinate units which should be their primary concern during peace-time operations, or else they can be reassigned to jobs where they can be more productive. The numerous positions of radio, telephone, and switchboard operators can be utilized in other equally important areas, such as perimeter defense and maintenance.

Mathematical programming reduces the amount of coordination time needed between commanders and staff sections at different levels prior to and during the exercise. Early warning and a complete description of the mission statement can be relayed over the computer viewscreen with graphical representation as well. Extensive manual message traffic, coding and decoding, which consumes a great deal of time and manpower, is also avoided. Time and distance will also be saved by task organizing and assigning missions based on the shortest-distance program. As a result, units will have the best possible chance of providing the most extensive coverage, and doing it in the least amount of time as well.

Compared to present Air Defense operations, this new approach can provide added flexibility to command and control operations. The decision-maker no longer needs to rely on updates and briefings to evaluate the situation. He will have a decision-maker's tool that is easily accessible via a portable computer and contains all the vital information he needs concerning the field exercise. The reduced staff and equipment required in the command and control center results in a much more flexible command post, easier to deploy, easier to relocate, and harder to detect. Flexibility is also generated when dealing with new objectives or changes in unit strengths. By weighting the objective function appropriately, the commander is provided a defense design which maximizes the air defense coverage based on current information, the highest priorities, and the best organization of his subordinate units.

5.2 Disadvantages

In addition to the advantages of using techniques of mathematical programming during a field exercise, there are several disadvantages to overcome before the benefits of mathematical modeling, network designs, and computer algorithms can be expanded upon and included into the Army inventory. This type of approach is new at unit level; it will challenge an established method of operation that military members are already familiar with, and it will involve the development of supporting activities.

This approach will meet with initial resistance, just because it is new. It will have to be tested in the field environment and explained in the classroom. A training program will have to be implemented involving an explanation of the methodology and a hands-on phase for military personnel to become familiar with the equipment and mathematical programming capabilities.

This approach will challenge an existing system of command and control. If it is implemented it will result in a smaller staff for command and control operations. Reductions like this usually meet with a great deal of resistance, particularly from those who are in command. A direct result of smaller staffs will be greater competition for fewer jobs. This can be taken as an advantage or a disadvantage, depending on your viewpoint.

Another area of concern is the composition of supporting activities to complement this new approach. To take advantage of the mathematical programming capabilities, a flexible computer software system will have to be developed including a complete data base structure with linear and integer programming solution algorithms and graphical representations. To utilize this computer oriented approach, signal capabilities will have to be able to provide the necessary communication links.

CHAPTER 6

RESULTS AND RECOMMENDATIONS

The results and recommendations of this paper are explained in terms of program validation, other areas for mathematical programming implementation, and concluding remarks.

6.1 Validation

Twelve objectives in Europe were used for the validation of the mathematical models along with realistic road distances, travel times, and coverage requirements. Three models [Appendices 1, 2, and 3] were tested and solutions [Appendices 1, 2, and 3] were generated using a linear and integer programming algorithm (see [20] and [21]) to check the applicability of mathematical programming during a field exercise.

The linear programming problem from Phase 2, establishing optimal coverage requirements at Group level, was easily solved and the results were consistent with the weighted priority scheme. The real value of the linear model is realized when there are numerous objectives with a total number of available weapon systems well below the sum of the upper bound requirements. This can be explained by a short review of an initial model tested with 10 objectives and 72 available Chaparral systems, and the final formulation [Appendix 1] with 12 objectives and only 66 Chaparral systems. In the initial case, the total number of Chaparrals needed to meet all upper bound coverage requirements was 75. This meant almost all objectives, except the last one or two on the priority scale, were assigned the upper bound number of Chaparrals. There was not much need in checking upper and lower bound constraints for an optimal coverage assignment. When 12 objectives, with a total upper bound requirement of

85 Chaparrals and a lower bound of 55 were used, a not so obvious optimal solution was quickly generated using a linear programming model. These results were used along with distance, time, and coverage matrices (Phase 3), to provide the necessary inputs to the shortest-distance model of Phase 4.

The integer programming problem of Phase 4 at Group level provided the first real insight to the need of possible suboptimization. Using the coverage requirements of the linear programming solution, demand exactly equals the potential supply of weapon systems, as long as the sum of upper bound requirements exceeds the total number of weapon systems available. With only one supply node (Phase 2), the source and assignment of weapon systems equaling the demand was not a problem. When there are three source nodes, as in Phase 4 (A,B,C), each with a certain number of systems to supply and only Battery size transfers (12 weapons) allowed, an optimal integer solution requires a considerable amount of time (over 4100 iterations and 422 branch and bounds on a Prime 850). To offset this excessive amount of time, Redeye weapon systems were included in the Battalions' supply base (4 Redeye teams at each Battalion). Using this method, a solution consisting of minimum distance, restructured units, and the allocation of the optimal number of weapons for each objective took only 81 iterations and 12 branch and bounds. The lesson to be learned from this model was in the relationship of supply and demand. With strict equality, excessive computational time or suboptimization occurs (some of the optimal coverage requirements may not be satisfied). The solution is to expand the supply base by incorporating the Redeye systems which are controlled at the Battery/Battalion level, or to relax some of the coverage requirements by allowing an exceptable range of weapon system coverage for selected (lowest priority) objectives.

The results of the Phase 4 solution (Appendix 2) were used as input for the shortest-distance problem of Phase 5 at the Battalion level. One of the three Battalions (C) was used to illustrate the mathematical programming technique. The Battalion had 4 objectives assigned to it with an excess of 6 Chaparrals and 3 Vulcans (supply > demand). An optimal solution was quickly computed and checked for feasibility. If strict equality applied between supply and demand, the flexibility, as discussed for Phase 4, would have been required presenting the possibility of a suboptimal solution for weapons' assignment at the Battery level. The aid of using an integer program at this level can be emphasized by examining the composition and assignments of Battalion A. Battalion C, with 4 Batteries and 4 objectives, provided a relatively straightforward problem. Given a Battalion with 4 or 5 Batteries and 6 or more objectives at many different locations causes the scheduling and assignment of weapon systems and objectives to become very complex without the help of a decision-making tool.

The mathematical models provided very realistic and appropriate results. Insight, concerning solution time, strict equality between supply and demand, and the possibility of suboptimization at lower levels of command and control (Battalion and Battery) provided useful information for minor modifications of the original formulations.

6.2 Other Areas for Implementation

There are a variety of areas, or situations, in which techniques of mathematical programming can be successfully and beneficially applied for military purposes. Two of these areas are wargaming and support activities.

6.2.1 Wargaming:

There are many types of wargaming models in many of the military service schools. They involve interaction between different teams or opposing forces and among staff sections or entire command and control elements. An interactive computer programming system can be developed, which incorporates many concepts of mathematical programming, to provide very realistic and easy to operate wargaming models.

In this game theory situation, two sides are actively working against each other. For Air Defense purposes, this involves an offensive element attacking the Air Defense forces or the critical objectives they are trying to protect. As illustrated in this paper, analytic modeling, dynamic programming, network design, and linear and integer programming have been used to develop the defensive decision-making tool. There have also been recent efforts by members of Lincoln Labs of MIT [17] to design and employ optimal weapons allocation against layered defenses. An abstract of that work follows:

> We solve the problem of optimal allocation of (offensive) weapons to targets in the presence of layered regional defenses. The general solution technique is an integer program transformable to a minimum cost network flow. This model assumes exhaustion of defenses. Results of a small sample scenario are included. Additionally, a representative attrition algorithm is described and the two models combined to form a hybrid algorithm is described. The hybrid algorithm allows for leakage through defenses while maintaining a feasible allocation scheme.

Coordination of these types of efforts can result in a very modern, in terms of operations research methods, approach for command and control operations.

6.2.2 Support Activities:

Developing a transportable computer system for Air Defense command and control purposes provides a useful example for similar efforts in other branches of the Army, and service departments of the Armed Forces. One of these branches, which provides communication support for Air Defense units during field exercises, is the Signal Corps. Mathematical programming can aid some of their objectives of:

1. providing communications capability to multiple users at multiple locations;
2. developing efficient communications networks;
3. assigning scarce Signal equipment to appropriate communication centers; and
4. reacting to new communications requirements as the exercise develops.

These types of objectives are involved not only in the Air Defense or Signal branches, but in many different types of military elements and phases of military operations.

6.3 Conclusion

Modern techniques of mathematical programming can play an important role in improving command and control of Air Defense units during a field exercise. There are many advantages of using mathematical programming techniques, including optimal allocation and scheduling of resources, but there are hurdles to overcome in terms of testing and training military personnel in these areas, before the military takes advantage of such modern techniques. The military applications of mathematical programming, to-date, have concentrated on strategy evaluation at upper echelons of the hierarchy, with relatively few applications dealing with strategy generation for improved field unit operations. Part of the explanation may have been the inability to use mobile computer capabilities or the inability to develop a useful and interactive software system. With modern computer hardware and software capabilities, this is no longer the case. A useful decision-making tool for Air Defense commanders can be developed and lead to many related areas of application throughout the Armed Forces.

APPEDDIX I

Phase 2 Model and Solution

```
MAX      100 C1 + 80 C2 + 60 C3 + 40 C4 + 20 C5 + 90 C6 + 70 C7
     + 50 C8 + 30 C9 + 10 C10 + 65 C11 + 45 C12 + 100 V1 + 80 V2
     + 60 V3 + 40 V4 + 20 V5 + 90 V6 + 70 V7 + 50 V8 + 30 V9
     + 10 V10 + 65 V11 + 45 V12
SUBJECT TO
    2)      C1 +  SC1 =     8
    3)      C2 +  SC2 =     6
    4)      C3 +  SC3 =     8
    5)      C4 +  SC4 =     6
    6)      C5 +  SC5 =     8
    7)      C6 +  SC6 =     7
    8)      C7 +  SC7 =     7
    9)      C8 +  SC8 =     6
   10)      C9 +  SC9 =     8
   11)      C10 +  SC10 =     8
   12)      C11 +  SC11 =     7
   13)      C12 +  SC12 =     6
   14)      SC1 <=    3
   15)      SC2 <=    2
   16)      SC3 <=    3
   17)      SC4 <=    2
   18)      SC5 <=    4
   19)      SC6 <=    3
   20)      SC7 <=    3
   21)      SC8 <=    2
   22)      SC9 <=    2
   23)      SC10 <=    4
   24)      SC11 <=    3
   25)      SC12 <=    2
   26)      V1 +  SV1 =     8
   27)      V2 +  SV2 =     6
   28)      V3 +  SV3 =     7
   29)      V4 +  SV4 =     7
   30)      V5 +  SV5 =     8
   31)      V6 +  SV6 =     8
   32)      V7 +  SV7 =     6
   33)      V8 +  SV8 =     8
   34)      V9 +  SV9 =     8
   35)      V10 +  SV10 =     7
   36)      V11 +  SV11 =     7
   37)      V12 +  SV12 =     8
   38)      SV1 <=    4
   39)      SV2 <=    4
   40)      SV3 <=    3
   41)      SV4 <=    3
   42)      SV5 <=    4
   43)      SV6 <=    4
   44)      SV7 <=    2
   45)      SV8 <=    4
   46)      SV9 <=    4
   47)      SV10 <=    2
   48)      SV11 <=    3
   49)      SV12 <=    4
   50)      C1 +  C2 +  C3 +  C4 +  C5 +  C6 +  C7 +  C8 +  C9 +  C10
     +  C11 +  C12 <=    66
   51)      V1 +  V2 +  V3 +  V4 +  V5 +  V6 +  V7 +  V8 +  V9 +  V10
     +  V11 +  V12 <=    66
END
```

Phase 2

```
: GO
   LP OPTIMUM FOUND  AT STEP     38

        OBJECTIVE FUNCTION VALUE
 1)          8020.00000
```

VARIABLE	VALUE	REDUCED COST
C1	8.000000	0.000000
C2	6.000000	0.000000
C3	5.000000	0.000000
C4	4.000000	0.000000
C5	4.000000	0.000000
C6	7.000000	0.000000
C7	7.000000	0.000000
C8	4.000000	0.000000
C9	6.000000	0.000000
C10	4.000000	0.000000
C11	7.000000	0.000000
C12	4.000000	0.000000
V1	8.000000	0.000000
V2	6.000000	0.000000
V3	6.000000	0.000000
V4	4.000000	0.000000
V5	4.000000	0.000000
V6	8.000000	0.000000
V7	6.000000	0.000000
V8	4.000000	0.000000
V9	4.000000	0.000000
V10	5.000000	0.000000
V11	7.000000	0.000000
V12	4.000000	0.000000
SC1	0.000000	40.000000
SC2	0.000000	20.000000
SC3	3.000000	0.000000
SC4	2.000000	0.000000
SC5	4.000000	0.000000
SC6	0.000000	30.000000
SC7	0.000000	10.000000
SC8	2.000000	0.000000
SC9	2.000000	0.000000
SC10	4.000000	0.000000
SC11	0.000000	5.000000
SC12	2.000000	0.000000
SV1	0.000000	40.000000
SV2	0.000000	20.000000
SV3	1.000000	0.000000
SV4	3.000000	0.000000
SV5	4.000000	0.000000
SV6	0.000000	30.000000
SV7	0.000000	10.000000
SV8	4.000000	0.000000
SV9	4.000000	0.000000
SV10	2.000000	0.000000
SV11	0.000000	5.000000
SV12	4.000000	0.000000

Phase 2

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	40.000000
3)	0.000000	20.000000
4)	0.000000	0.000000
5)	0.000000	-20.000000
6)	0.000000	-40.000000
7)	0.000000	30.000000
8)	0.000000	10.000000
9)	0.000000	-10.000000
10)	0.000000	-30.000000
11)	0.000000	-50.000000
12)	0.000000	5.000000
13)	0.000000	-15.000000
14)	3.000000	0.000000
15)	2.000000	0.000000
16)	0.000000	0.000000
17)	0.000000	20.000000
18)	0.000000	40.000000
19)	3.000000	0.000000
20)	3.000000	0.000000
21)	0.000000	10.000000
22)	0.000000	30.000000
23)	0.000000	50.000000
24)	3.000000	0.000000
25)	0.000000	15.000000
26)	0.000000	40.000000
27)	0.000000	20.000000
28)	0.000000	0.000000
29)	0.000000	-20.000000
30)	0.000000	-40.000000
31)	0.000000	30.000000
32)	0.000000	10.000000
33)	0.000000	-10.000000
34)	0.000000	-30.000000
35)	0.000000	-50.000000
36)	0.000000	5.000000
37)	0.000000	-15.000000
38)	4.000000	0.000000
39)	4.000000	0.000000
40)	2.000000	0.000000
41)	0.000000	20.000000
42)	0.000000	40.000000
43)	4.000000	0.000000
44)	2.000000	0.000000
45)	0.000000	10.000000
46)	0.000000	30.000000
47)	0.000000	50.000000
48)	3.000000	0.000000
49)	0.000000	15.000000
50)	0.000000	60.000000
51)	0.000000	60.000000

NO. ITERATIONS= 38

Phase 2

DO RANGE(SENSITIVITY) ANALYSIS?
? YES

RANGES IN WHICH THE BASIS IS UNCHANGED

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
C1	100.000000	INFINITY	40.000000
C2	80.000000	INFINITY	20.000000
C3	60.000000	5.000000	10.000000
C4	40.000000	20.000000	INFINITY
C5	20.000000	40.000000	INFINITY
C6	90.000000	INFINITY	30.000000
C7	70.000000	INFINITY	10.000000
C8	50.000000	10.000000	INFINITY
C9	30.000000	30.000000	INFINITY
C10	10.000000	50.000000	INFINITY
C11	65.000000	INFINITY	5.000000
C12	45.000000	15.000000	INFINITY
V1	100.000000	INFINITY	40.000000
V2	80.000000	INFINITY	20.000000
V3	60.000000	5.000000	10.000000
V4	40.000000	20.000000	INFINITY
V5	20.000000	40.000000	INFINITY
V6	90.000000	INFINITY	30.000000
V7	70.000000	INFINITY	10.000000
V8	50.000000	10.000000	INFINITY
V9	30.000000	30.000000	INFINITY
V10	10.000000	50.000000	INFINITY
V11	65.000000	INFINITY	5.000000
V12	45.000000	15.000000	INFINITY
SC1	0.000000	40.000000	INFINITY
SC2	0.000000	20.000000	INFINITY
SC3	0.000000	10.000000	5.000000
SC4	0.000000	INFINITY	20.000000
SC5	0.000000	INFINITY	40.000000
SC6	0.000000	30.000000	INFINITY
SC7	0.000000	10.000000	INFINITY
SC8	0.000000	INFINITY	10.000000
SC9	0.000000	INFINITY	30.000000
SC10	0.000000	INFINITY	50.000000
SC11	0.000000	5.000000	INFINITY
SC12	0.000000	INFINITY	15.000000
SV1	0.000000	40.000000	INFINITY
SV2	0.000000	20.000000	INFINITY
SV3	0.000000	10.000000	5.000000
SV4	0.000000	INFINITY	20.000000
SV5	0.000000	INFINITY	40.000000
SV6	0.000000	30.000000	INFINITY
SV7	0.000000	10.000000	INFINITY
SV8	0.000000	INFINITY	10.000000
SV9	0.000000	INFINITY	30.000000
SV10	0.000000	INFINITY	50.000000
SV11	0.000000	5.000000	INFINITY
SV12	0.000000	INFINITY	15.000000

Phase 2

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	8.000000	0.000000	3.000000
3	6.000000	0.000000	3.000000
4	8.000000	0.000000	3.000000
5	6.000000	0.000000	3.000000
6	8.000000	0.000000	3.000000
7	7.000000	0.000000	3.000000
8	7.000000	0.000000	3.000000
9	6.000000	0.000000	3.000000
10	8.000000	0.000000	3.000000
11	8.000000	0.000000	3.000000
12	7.000000	0.000000	3.000000
13	6.000000	0.000000	3.000000
14	3.000000	INFINITY	3.000000
15	2.000000	INFINITY	2.000000
16	3.000000	INFINITY	0.000000
17	2.000000	3.000000	0.000000
18	4.000000	3.000000	0.000000
19	3.000000	INFINITY	3.000000
20	3.000000	INFINITY	3.000000
21	2.000000	3.000000	0.000000
22	2.000000	3.000000	0.000000
23	4.000000	3.000000	0.000000
24	3.000000	INFINITY	3.000000
25	2.000000	3.000000	0.000000
26	8.000000	2.000000	1.000000
27	6.000000	2.000000	1.000000
28	7.000000	2.000000	1.000000
29	7.000000	2.000000	1.000000
30	8.000000	2.000000	1.000000
31	8.000000	2.000000	1.000000
32	6.000000	2.000000	1.000000
33	8.000000	2.000000	1.000000
34	8.000000	2.000000	1.000000
35	7.000000	2.000000	1.000000
36	7.000000	2.000000	1.000000
37	8.000000	2.000000	1.000000
38	4.000000	INFINITY	4.000000
39	4.000000	INFINITY	4.000000
40	3.000000	INFINITY	2.000000
41	3.000000	1.000000	2.000000
42	4.000000	1.000000	2.000000
43	4.000000	INFINITY	4.000000
44	2.000000	INFINITY	2.000000
45	4.000000	1.000000	2.000000
46	4.000000	1.000000	2.000000
47	2.000000	1.000000	2.000000
48	3.000000	INFINITY	3.000000
49	4.000000	1.000000	2.000000
50	66.000000	3.000000	0.000000
51	66.000000	1.000000	2.000000

APPENDIX 2

Phase 4 Model and Solution

```
MIN      10 AC1 + 5 AC2 + 15 AC3 + 10 AC4 + 30 AC5 + 35 AC6
     + 40 AC7 + 50 AC8 + 45 AC9 + 35 AC10 + 40 AC11 + 20 AC12
     + 10 AV1 + 5 AV2 + 15 AV3 + 10 AV4 + 30 AV5 + 35 AV6 + 40 AV7
     + 50 AV8 + 45 AV9 + 35 AV10 + 40 AV11 + 20 AV12 + 30 BC1
     + 25 BC2 + 28 BC3 + 40 BC4 + 50 BC5 + 55 BC6 + 70 BC7 + 80 BC8
     + 75 BC9 + 5 BC10 + 10 BC11 + 50 BC12 + 30 BV1 + 25 BV2
     + 28 BV3 + 40 BV4 + 50 BV5 + 55 BV6 + 70 BV7 + 80 BV8 + 75 BV9
     + 5 BV10 + 10 BV11 + 50 BV12 + 40 CC1 + 45 CC2 + 50 CC3
     + 25 CC4 + 60 CC5 + 65 CC6 + 5 CC7 + 10 CC8 + 20 CC9 + 70 CC10
     + 75 CC11 + 50 CC12 + 40 CV1 + 45 CV2 + 50 CV3 + 25 CV4
     + 60 CV5 + 65 CV6 + 5 CV7 + 10 CV8 + 20 CV9 + 70 CV10
     + 75 CV11 + 50 CV12 + 30 CBA + 40 CCA + 30 CAB + 40 CAC
     + 30 VBA + 40 VCA + 30 VAB + 40 VAC + 70 CCB + 70 CBC + 70 VCB
     + 70 VBC
SUBJECT TO
    2)  - 8 AC1 - 6 AC2 - 5 AC3 - 4 AC4 - 4 AC5 - 7 AC6 - 7 AC7
     - 4 AC8 - 6 AC9 - 4 AC10 - 7 AC11 - 4 AC12 +  AC =    0
    3)  - 8 AV1 - 6 AV2 - 6 AV3 - 4 AV4 - 4 AV5 - 8 AV6 - 6 AV7
     - 4 AV8 - 4 AV9 - 5 AV10 - 7 AV11 - 4 AV12 +  AV =    0
    4)  - 8 BC1 - 6 BC2 - 5 BC3 - 4 BC4 - 4 BC5 - 7 BC6 - 7 BC7
     - 4 BC8 - 6 BC9 - 4 BC10 - 7 BC11 - 4 BC12 +  BC =    0
    5)  - 8 BV1 - 6 BV2 - 6 BV3 - 4 BV4 - 4 BV5 - 8 BV6 - 6 BV7
     - 4 BV8 - 4 BV9 - 5 BV10 - 7 BV11 - 4 BV12 +  BV =    0
    6)  - 8 CC1 - 6 CC2 - 5 CC3 - 4 CC4 - 4 CC5 - 7 CC6 - 7 CC7
     - 4 CC8 - 6 CC9 - 4 CC10 - 7 CC11 - 4 CC12 +  CC =    0
    7)  - 8 CV1 - 6 CV2 - 6 CV3 - 4 CV4 - 4 CV5 - 8 CV6 - 6 CV7
     - 4 CV8 - 4 CV9 - 5 CV10 - 7 CV11 - 4 CV12 +  CV =    0
    8)     CAB +  CAC <=    1
    9)     CBA +  CCA <=    1
   10)     VAB +  VAC <=    1
   11)     VBA +  VCA <=    1
   12)     CBA +  CBC <=    1
   13)     CAB +  CCB <=    1
   14)     VBA +  VBC <=    1
   15)     VAB +  VCB <=    1
   16)     CCA +  CCB <=    1
   17)     CAC +  CCB <=    1
   18)     VCA +  VCB <=    1
   19)     VAC +  VBC <=    1
   20)     AC1 +  BC1 +   CC1 =     1
   21)     AC2 +  BC2 +   CC2 =     1
   22)     AC3 +  BC3 +   CC3 =     1
   23)     AC4 +  BC4 +   CC4 =     1
   24)     AC5 +  BC5 +   CC5 =     1
   25)     AC6 +  BC6 +   CC6 =     1
   26)     AC7 +  BC7 +   CC7 =     1
   27)     AC8 +  BC8 +   CC8 =     1
   28)     AC9 +  BC9 +   CC9 =     1
   29)     AC10 +  BC10 +   CC10 =     1
   30)     AC11 +  BC11 +   CC11 =     1
   31)     AC12 +  BC12 +   CC12 =     1
```

Phase 4

```
32)      AV1 +  BV1 +  CV1 =     1
33)      AV2 +  BV2 +  CV2 =     1
34)      AV3 +  BV3 +  CV3 =     1
35)      AV4 +  BV4 +  CV4 =     1
36)      AV5 +  BV5 +  CV5 =     1
37)      AV6 +  BV6 +  CV6 =     1
38)      AV7 +  BV7 +  CV7 =     1
39)      AV8 +  BV8 +  CV8 =     1
40)      AV9 +  BV9 +  CV9 =     1
41)      AV10 +  BV10 +  CV10 =     1
42)      AV11 +  BV11 +  CV11 =     1
43)      AV12 +  BV12 +  CV12 =     1
44)      AC1 -  AV1 =    0
45)      AC2 -  AV2 =    0
46)      AC3 -  AV3 =    0
47)      AC4 -  AV4 =    0
48)      AC5 -  AV5 =    0
49)      AC6 -  AV6 =    0
50)      AC7 -  AV7 =    0
51)      AC8 -  AV8 =    0
52)      AC9 -  AV9 =    0
53)      AC10 -  AV10 =    0
54)      AC11 -  AV11 =    0
55)      AC12 -  AV12 =    0
56)      BC1 -  BV1 =    0
57)      BC2 -  BV2 =    0
58)      BC3 -  BV3 =    0
59)      BC4 -  BV4 =    0
60)      BC5 -  BV5 =    0
61)      BC6 -  BV6 =    0
62)      BC7 -  BV7 =    0
63)      BC8 -  BV8 =    0
64)      BC9 -  BV9 =    0
65)      BC10 -  BV10 =    0
66)      BC11 -  BV11 =    0
67)      BC12 -  BV12 =    0
68)      CC1 -  CV1 =    0
69)      CC2 -  CV2 =    0
70)      CC3 -  CV3 =    0
71)      CC4 -  CV4 =    0
72)      CC5 -  CV5 =    0
73)      CC6 -  CV6 =    0
74)      CC7 -  CV7 =    0
75)      CC8 -  CV8 =    0
76)      CC9 -  CV9 =    0
77)      CC10 -  CV10 =    0
78)      CC11 -  CV11 =    0
79)      CC12 -  CV12 =    0
80)  - 12 VBA - 12 VCA + 12 VAB + 12 VAC +  AV +  SAV =     24
81)  - 12 CBA - 12 CCA + 12 CAB + 12 CAC +  AC +  SAC =     24
82)    12 CBA - 12 CAB - 12 CCB + 12 CBC +  BC +  SBC =     24
83)    12 VBA - 12 VAB - 12 VCB + 12 VBC +  BV +  SBV =     24
84)    12 CCA - 12 CAC + 12 CCB - 12 CBC +  CC +  SCC =     24
85)    12 VCA - 12 VAC + 12 VCB - 12 VBC +  CV +  SCV =     24
END
INTEGER-VARIABLES=      84
```

Phase 4

```
GO
   LP OPTIMUM FOUND  AT STEP     49

         OBJECTIVE FUNCTION VALUE
1)          427.666687
```

VARIABLE	VALUE	REDUCED COST
AC1	1.000000	0.000000
AC2	1.000000	0.000000
AC3	0.000000	1.500000
AC4	1.000000	0.000000
AC5	1.000000	-20.000000
AC6	1.000000	0.000000
AC7	0.000000	113.333313
AC8	0.000000	0.000000
AC9	0.000000	0.000000
AC10	0.000000	82.500000
AC11	0.000000	0.000000
AC12	1.000000	-33.333344
AV1	1.000000	0.000000
AV2	1.000000	-10.000000
AV3	0.000000	0.000000
AV4	1.000000	0.000000
AV5	1.000000	0.000000
AV6	1.000000	-2.499992
AV7	0.000000	0.000000
AV8	0.000000	33.333328
AV9	0.000000	0.000000
AV10	0.000000	0.000000
AV11	0.000000	95.000000
AV12	1.000000	0.000000
BC1	0.000000	0.000000
BC2	0.000000	0.000000
BC3	1.000000	0.000000
BC4	0.000000	39.999992
BC5	0.000000	0.000000
BC6	0.000000	0.000000
BC7	0.000000	140.833313
BC8	0.000000	0.000000
BC9	0.000000	35.000000
BC10	1.000000	0.000000
BC11	1.000000	0.000000
BC12	0.000000	6.666656
BV1	0.000000	0.000000
BV2	0.000000	0.000000
BV3	1.000000	0.000000
BV4	0.000000	0.000000
BV5	0.000000	0.000000
BV6	0.000000	0.000000
BV7	0.000000	0.000000
BV8	0.000000	73.333328
BV9	0.000000	0.000000
BV10	1.000000	0.000000
BV11	1.000000	0.000000
BV12	0.000000	0.000000

Phase 4

CC1	0.000000	6.666672
CC2	0.000000	30.000008
CC3	0.000000	34.833344
CC4	0.000000	3.333336
CC5	0.000000	0.000000
CC6	0.000000	0.000000
CC7	1.000000	0.000000
CC8	1.000000	-73.333328
CC9	1.000000	-83.333313
CC10	0.000000	0.000000
CC11	0.000000	0.000000
CC12	0.000000	0.000000
CV1	0.000000	0.000000
CV2	0.000000	0.000000
CV3	0.000000	0.000000
CV4	0.000000	0.000000
CV5	0.000000	13.333344
CV6	0.000000	7.500015
CV7	1.000000	0.000000
CV8	1.000000	0.000000
CV9	1.000000	0.000000
CV10	0.000000	122.500000
CV11	0.000000	118.333344
CV12	0.000000	0.000000
CBA	0.666667	0.000000
CCA	0.083333	0.000000
CAB	0.000000	59.999985
CAC	0.000000	79.999985
VBA	0.500000	0.000000
VCA	0.333333	0.000000
VAB	0.000000	59.999985
VAC	0.000000	79.999985
CCB	0.000000	60.000000
CBC	0.000000	79.999985
VCB	0.000000	60.000000
VBC	0.000000	79.999985
AC	33.000000	0.000000
AV	34.000000	0.000000
BC	16.000000	0.000000
BV	18.000000	0.000000
CC	17.000000	0.000000
CV	14.000000	0.000000
SAV	0.000000	3.333333
SAC	0.000000	3.333333
SBC	0.000000	0.833333
SBV	0.000000	0.833333
SCC	6.000000	0.000000
SCV	6.000000	0.000000

```
NO. ITERATTONS=        49
 BRANCHES=     0 DETERM.= -2.071E  4
   FIX ALL VARS.(    29)  WITH RC >    3.33334
SET       CBA TO     0 AT     1 BND=  -429.33337     TWIN=  -427.66669
SET       AC1 TO     0 AT     2 BND=  -429.55560     TWIN=  -431.33337
SET       AC3 TO     1 AT     3 BND=  -431.66669     TWIN=  -436.66675
SET       AC6 TO     1 AT     4 BND=  -433.33337     TWIN=  -436.66669
SET       CCA TO     0 AT     5 BND= -0.99999986E 30 TWIN= -0.99999986E 30
 DELETE        CCA AT LEVEL      5
   FLIP        AC6 TO       0 WITH BOUND  -436.6667
 DELETE        AC6 AT LEVEL      4
   FLIP        AC3 TO       0 WITH BOUND  -436.6667
 DELETE        AC3 AT LEVEL      3
```

Phase 4

NEW INTEGER SOLUTION OF 440.000 AT BRANCH 10 PIVOT 81

OBJECTIVE FUNCTION VALUE

1) 440.000000

VARIABLE	VALUE	REDUCED COST
AC1	1.000000	0.000000
AC2	1.000000	0.000000
AC3	1.000000	4.000000
AC4	0.000000	-6.666664
AC5	1.000000	-19.999992
AC6	1.000000	0.000000
AC7	0.000000	105.000000
AC8	0.000000	0.000000
AC9	0.000000	0.000000
AC10	0.000000	84.999985
AC11	0.000000	0.000000
AC12	1.000000	-36.666664
AV1	1.000000	0.000000
AV2	1.000000	-10.000004
AV3	1.000000	0.000000
AV4	0.000000	0.000000
AV5	1.000000	0.000000
AV6	1.000000	0.000000
AV7	0.000000	0.000000
AV8	0.000000	33.333328
AV9	0.000000	0.000000
AV10	0.000000	0.000000
AV11	0.000000	94.999969
AV12	1.000000	0.000000
BC1	0.000000	0.000000
BC2	0.000000	0.000000
BC3	0.000000	0.000000
BC4	0.000000	33.333336
BC5	0.000000	0.000000
BC6	0.000000	0.000000
BC7	0.000000	135.000000
BC8	0.000000	0.000000
BC9	0.000000	40.000000
BC10	1.000000	0.000000
BC11	1.000000	0.000000
BC12	0.000000	3.333336
BV1	0.000000	0.000000
BV2	0.000000	0.000000
BV3	0.000000	0.000000
BV4	0.000000	0.000000
BV5	0.000000	0.000000
BV6	0.000000	0.000000
BV7	0.000000	0.000000
BV8	0.000000	73.333328
BV9	0.000000	0.000000
BV10	1.000000	0.000000
BV11	1.000000	0.000000
BV12	0.000000	0.000000

Phase 4

CC1	0.000000	13.333328
CC2	0.000000	34.999992
CC3	0.000000	39.000000
CC4	1.000000	0.000000
CC5	0.000000	0.000000
CC6	0.000000	0.000000
CC7	1.000000	0.000000
CC8	1.000000	-70.000000
CC9	1.000000	-73.333328
CC10	0.000000	0.000000
CC11	0.000000	0.000000
CC12	0.000000	0.000000
CV1	0.000000	0.000000
CV2	0.000000	0.000000
CV3	0.000000	0.000000
CV4	1.000000	0.000000
CV5	0.000000	16.666672
CV6	0.000000	13.333328
CV7	1.000000	0.000000
CV8	1.000000	0.000000
CV9	1.000000	0.000000
CV10	0.000000	125.833328
CV11	0.000000	124.166656
CV12	0.000000	0.000000
CBA	1.000000	30.000000
CCA	0.000000	40.000000
CAB	0.000000	30.000000
CAC	0.000000	40.000000
VBA	1.000000	0.000000
VCA	0.000000	0.000000
VAB	0.000000	89.999985
VAC	0.000000	110.000000
CCB	0.000000	70.000000
CBC	0.000000	70.000000
VCB	0.000000	59.999992
VBC	0.000000	80.000000
AC	34.000000	0.000000
AV	36.000000	0.000000
BC	11.000000	0.000000
BV	12.000000	0.000000
CC	21.000000	0.000000
CV	18.000000	0.000000
SAV	0.000000	5.833334
SAC	2.000000	0.000000
SBC	1.000000	0.000000
SBV	0.000000	0.833334
SCC	3.000000	0.000000
SCV	6.000000	0.000000

Phase 4

```
NO. ITERATIONS=        81
 BRANCHES=    10 DETERM.= 96.000E  0
 BOUND ON OPTIMUM:  427.6667
 DELETE        AC4 AT LEVEL       3
 DELETE        AC3 AT LEVEL       2
 DELETE        CBA AT LEVEL       1
   RELEASE FIXED VARS.
   FIX ALL VARS.(    31)  WITH RC >    1.37479
SET       AC1  TO     0 AT     1 BND=  -441.83337      TWIN=  -440.16711
 DELETE        AC1 AT LEVEL       1
   RELEASE FIXED VARS.
   FIX ALL VARS.(    30)  WITH RC >  0.624978
SET       AC2  TO     0 AT     1 BND=  -440.00000      TWIN=  -440.00031
 DELETE        AC2 AT LEVEL       1
   RELEASE FIXED VARS.
    ENUMERATION COMPLETE. BRANCHES=    12  PIVOTS=   122

LAST INTEGER SOLUTION IS THE BEST FOUND
: QUIT
```

APPENDIX 3

Phase 5 Model and Solution

```
     ALL

MIN      25 CCA4 + 5 CCA7 + 10 CCA8 + 20 CCA9 + 35 CCB4 + 10 CCB7
     + 15 CCB8 + 5 CCB9 + 25 VCC4 + 5 VCC7 + 10 VCC8 + 20 VCC9
     + 35 VCD4 + 10 VCD7 + 15 VCD8 + 5 VCD9 + 5 RA + 10 RB
SUBJECT TO
    2)  - 4 CCA4 - 7 CCA7 - 4 CCA8 - 6 CCA9 +  CCA =     0
    3)  - 4 CCB4 - 7 CCB7 - 4 CCB8 - 6 CCB9 +  CCB =     0
    4)  - 4 VCC4 - 6 VCC7 - 4 VCC8 - 4 VCC9 +  VCC =     0
    5)  - 4 VCD4 - 6 VCD7 - 4 VCD8 - 4 VCD9 +  VCD =     0
    6)  - 4 RA +  CCA +  SCCA =    10
    7)  - 4 RB +  CCB +  SCCB =    10
    8)     RA +  RB <=    1
    9)     4 VCCD - 4 VCDC +  VCC +  SVCC =     12
   10)  - 4 VCCD + 4 VCDC +  VCD +  SVCD =     12
   11)     CCA4 +  CCB4 =    1
   12)     CCA7 +  CCB7 =    1
   13)     CCA8 +  CCB8 =    1
   14)     CCA9 +  CCB9 =    1
   15)     VCC4 +  VCD4 =    1
   16)     VCC7 +  VCD7 =    1
   17)     VCC8 +  VCD8 =    1
   18)     VCC9 +  VCD9 =    1
END
INTEGER-VARIABLES=      20

: GO
    LP OPTIMUM FOUND  AT STEP     24

         OBJECTIVE FUNCTION VALUE
  1)          94.1071472
```

VARIABLE	VALUE	REDUCED COST
CCA4	1.000000	0.000000
CCA7	0.428571	0.000000
CCA8	1.000000	0.000000
CCA9	0.000000	19.285713
CCB4	0.000000	7.142853
CCB7	0.571429	0.000000
CCB8	0.000000	2.142857
CCB9	1.000000	0.000000
VCC4	1.000000	0.000000
VCC7	1.000000	0.000000
VCC8	1.000000	0.000000
VCC9	0.000000	15.000000
VCD4	0.000000	10.000000
VCD7	0.000000	5.000000
VCD8	0.000000	5.000000
VCD9	1.000000	0.000000
RA	0.250000	0.000000
RB	0.000000	7.857142
VCCD	0.000000	0.000000
VCDC	0.500000	0.000000
CCA	11.000000	0.000000
CCB	10.000000	0.000000
VCC	14.000000	0.000000
VCD	4.000000	0.000000
SCCA	0.000000	1.250000
SCCB	0.000000	0.535714
SVCC	0.000000	0.000000
SVCD	6.000000	0.000000

Phase 5

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	-1.250000
3)	0.000000	-0.535714
4)	0.000000	0.000000
5)	0.000000	0.000000
6)	0.000000	1.250000
7)	0.000000	0.535714
8)	0.750000	0.000000
9)	0.000000	0.000000
10)	0.000000	0.000000
11)	0.000000	-30.000000
12)	0.000000	-13.750000
13)	0.000000	-15.000000
14)	0.000000	-8.214285
15)	0.000000	-25.000000
16)	0.000000	-5.000000
17)	0.000000	-10.000000
18)	0.000000	-5.000000

```
NO. ITERATIONS=        24
 BRANCHES=     0 DETERM.= -1.120E  2
   FIX ALL VARS.(      5)  WITH RC >    5.00000
SET      CCA7 TO     1 AT     1 BND=  -96.250000     TWIN=  -96.250015
SET      CCA8 TO     0 AT     2 BND=  -96.250000     TWIN=  -100.00000
SET        RA TO     1 AT     3 BND=  -100.00000     TWIN=  -100.00000
SET      VCDC TO     1 AT     4 BND=  -100.00000     TWIN=  -100.00000
SET      VCCD TO     0 AT     5 BND=  -100.00000     TWIN=  -100.00000

NEW INTEGER SOLUTION OF    100.000     AT BRANCH      5 PIVOT     30

        OBJECTIVE FUNCTION VALUE
 1)         100.000000
```

VARIABLE	VALUE	REDUCED COST
CCA4	1.000000	0.000000
CCA7	1.000000	-5.000000
CCA8	0.000000	-5.000000
CCA9	0.000000	15.000000
CCB4	0.000000	10.000000
CCB7	0.000000	0.000000
CCB8	1.000000	0.000000
CCB9	1.000000	0.000000
VCC4	1.000000	0.000000
VCC7	1.000000	0.000000
VCC8	1.000000	0.000000
VCC9	0.000000	15.000000
VCD4	0.000000	10.000000
VCD7	0.000000	5.000000
VCD8	0.000000	5.000000
VCD9	1.000000	0.000000
RA	1.000000	5.000000
RB	0.000000	10.000000
VCCD	0.000000	0.000000
VCDC	1.000000	0.000000
CCA	11.000000	0.000000
CCB	10.000000	0.000000
VCC	14.000000	0.000000
VCD	4.000000	0.000000
SCCA	3.000000	0.000000
SCCB	0.000000	0.000000
SVCC	2.000000	0.000000
SVCD	4.000000	0.000000

Phase 5

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	0.000000
3)	0.000000	0.000000
4)	0.000000	0.000000
5)	0.000000	0.000000
6)	0.000000	0.000000
7)	0.000000	0.000000
8)	0.000000	0.000000
9)	0.000000	0.000000
10)	0.000000	0.000000
11)	0.000000	-25.000000
12)	0.000000	-10.000000
13)	0.000000	-15.000000
14)	0.000000	-5.000000
15)	0.000000	-25.000000
16)	0.000000	-5.000000
17)	0.000000	-10.000000
18)	0.000000	-5.000000

```
NO. ITERATIONS=        30
 BRANCHES=     5 DETERM.=  1.000E  0
 BOUND ON OPTIMUM:  94.10715
 DELETE      VCCD AT LEVEL      5
 DELETE      VCDC AT LEVEL      4
 DELETE        RA AT LEVEL      3
 DELETE      CCA8 AT LEVEL      2
   FLIP      CCA7 TO       0 WITH BOUND  -96.25002
 DELETE      CCA7 AT LEVEL      1
   RELEASE FIXED VARS.
    ENUMERATION COMPLETE. BRANCHES=      5  PIVOTS=     35

LAST INTEGER SOLUTION IS THE BEST FOUND
:
```

BIBLIOGRAPHY

Sources Cited in the Text

[1] "Advanced Defensive Fire Control Processing Methods Study." Defense Technical Information Center (DTIC). AD-C014047L. Alexandria, Virginia: Defense Logistics Agency, 1977.

[2] "Airborne Platforms Versus Airbreathing Strategic Threats." DTIC. AD-A09170. Alexandria, Virginia: Defense Logistics Agency, 1979.

[3] "An Analysis of MMCS Network Architectures to Support the Data Processing Requirements for Site Defense." DTIC. AD-A108589. Alexandria, Virginia: Defense Logistics Agency, 1980.

[4] "Analytic Methods for Studies of Command and Control in Anti-Ship Cruise Missile Defense." DTIC. AD-A052085. Alexandria, Virginia: Defense Logistics Agency, 1977.

[5] Bradley, Stephen P., Arnoldo C. Hax, and Thomas L. Magnanti, Applied Mathematical Programming. Reading, Massachusetts: Adison-Wesley Publishing Company, 1977.

[6] Claycombe, William W., and William G. Sullivan. Foundations of Mathematical Programming. Reston, Virginia: Reston Publishing Company, 1975.

[7] "The Complementary Programming Method for Solving Certain Non-Linear, Zero-One Programming Problems." DTIC. AD-880692. Alexandria, Virginia: Defense Logistics Agency, 1968.

[8] Cooper, Leon, and Mary W. Cooper. Introduction to Dynamic Programming. New York: Pergamon Press, 1981.

[9] "Dynamic System Mathematical Models for Roland II." DTIC. AD-C004188. Alexandria, Virginia: Defense Logistics Agency, 1975.

[10] "Marine Aviation Requirements Mathematical Programming Model (MARMAP) Description and Operating Instructions." DTIC. AD-B012794L. Alexandria, Virginia: Defense Logistics Agency, 1975.

[11] "Mapping Out Views of the World." Soldiers. March 1984. Volume 39, No. 3, p. 18-23.

[12] McMillan, Claude Jr. Mathematical Programming. New York: John Wiley and Sons, 1970.

[13] "MVADEM User's Guide and Reference Manual." DTIC. AD-B066016L. Alexandria, Virginia: Defense Logistics Agency, 1982.

[14] "Optimal Allocation of Tactical Missiles Between Valued Targets and Defense Targets." DTIC. AD-A110876. Alexandria, Virginia: Defense Logistics Agency, 1981.

[15] "Optimal SAM Defense System." DTIC. AD-717942. Alexandria, Virginia: Defense Logistics Agency, 1971.

[16] "Optimization of Hardened Facilities to Achieve System-Level Survivability Requirements." DTIC. AD-C030172. Alexandria, Virginia: Defense Logistics Agency, 1981.

[17] Orlin, D. L. H. "A Force Level Analysis of Strategic Air Battles." Project Report CMT-52. Lexington, Massachusetts: Lincoln Laboratories-M.I.T., 1984.

[18] "Program OPSTRA 6: Effectiveness of a Three-Layer Defense Against An Optimally Allocated Offense." DTIC. AD-B012794L. Alexandria, Virginia: Defense Logistics Agency, 1975.

[19] Ryan, Thomas A., Brian L. Joiner, and Barbara F. Ryan. MINITAB Student Handbook. Boston, Massachusetts: Duxbury Press, 1976.

[20] Schrage, Linus. Linear Programming Models with LINDO. Palo Alto, California: The Scientific Press, 1981.

[21] Schrage, Linus. User's Manual for LINDO. Palo Alto, California: The Scientific Press, 1981.

[22] Shapiro, Jeremy F. Mathematical Programming: Structures and Algorithms. New York: John Wiley and Sons, 1979.

General Sources

"Air Defense Artillery Employment, Chaparral/Vulcan." FM44-3. Fort Bliss, Texas: U. S. Army Air Defense School, May 1977.

"Air Defense Artillery Reference Handbook." FM44-1-2. Fort Bliss, Texas: Washington, D.C.: Headquarters Department of the Army, June 1978.

Anderson, David R., Dennis J. Sweeney, and Thomas A. Williams. An Introduction to Management Science. New York: West Publishing Company, 1982.

Harowitz, Ellis, and Sartaj Sahni. Fundamentals of Computer Algorithms. Rockville, Maryland: Computer Science Press, 1984.

Holzman, Albert G. Mathematical Programming for Operations Researchers and Computer Scientists. New York: Marcel Dekker, Inc., 1981.

Houp, Kenneth W., and Thomas E. Pearsall. Reporting Technical Information. New York: Macmillan Publishing Co., 1980.

Korte, Bernhard. Modern Applied Mathematics, Optimization and Operations Research. New York: North-Holland Publishing Company, 1982.

Krajewski, Lee J., and Howard E. Thompson. Management Science. New York: John Wiley and Sons, 1981.

Larson, Richard C., and Amedeo R. Odoni. Urban Operations Research. Englewood Cliffs, New Jersey: Prentice-Hall, 1981.

Meredith, Jack R., and Efraim Turban. Fundamentals of Management Science. Texas: Business Publications, 1981.

Michaelson, Herbert B. How to Write and Publish Engineering Papers and Reports. Philadelphia, Pennsylvania: ISI Press, 1982.

Smith, David K. Network Optimization Practice. New York: Ellis Horword Limited, 1982.

"U. S. Army Air Defense Artillery Employment." FM44-1. Fort Bliss, Texas: U. S. Army Air Defense School, March 1976.

END

FILMED

7-84

DTIC